# 14-moment maximum-entropy VDF: application to electrons in ExB fields

S. Boccelli,[1,2] F. Giroux,[1] T.E. Magin,[2] C.P.T. Groth,[3] J.G. McDonald[1]

(1) University of Ottawa, ON, Canada
(2) von Karman Institute for Fluid Dynamics, Belgium
(3) University of Toronto Institute for Aerospace Studies, ON, Canada


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# *14-moment maximum-entropy VDF: application to electrons in ExB fields (1 of 2)*

$$f_{14} = \exp\left(a + a_i v_i + a_{ij} v_i v_j + a_{i3} v^2 v_i + a_4 v^4\right), \quad i = (x, y, z)$$

Special cases:

$$\mathcal{M} = \exp(a)\exp(a_i v_i + a_2 v^2)$$

Maxwellian

$$\mathcal{D} = \exp(-a v^4)$$

Druyvesteyn VDF

These introduce:
- Anisotropy
- Asymmetry
- Non-Maxwellian kurtosis

Advantages:
- Positive by construction
- Results in a hyperbolic system of PDEs

Disadvantages:
- The coefficients "a, a_i, ..." in the VDF are an unknown function of the moments (numerical iterations/approx)
- Junk subspace (∃ singularity in moment space)

Resulting sys of hyperbolic PDEs:

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{F}_i}{\partial x_i} = \vec{S}$$

$$\vec{U} = \begin{pmatrix} \rho & \text{1 eq.} \\ \rho u_i & \text{3 eqs.} \\ \rho u_i u_j + P_{ij} & \text{6 eqs.} \\ \rho u^2 u_i + \cdots + q_i & \text{3 eqs.} \\ \rho u^4 + \cdots + R & \text{1 eq.} \end{pmatrix}$$

References:

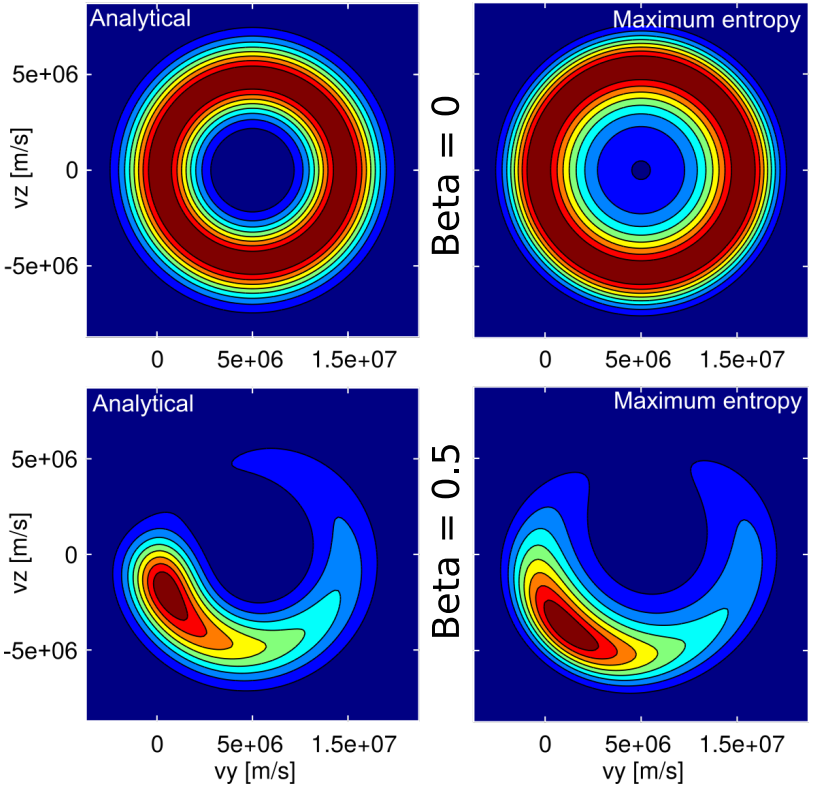

- Levermore, *Moment closure hierarchies for kinetic theories*, J. Stat. Phys, 1996.
- Muller & Ruggeri, *Extended Thermodynamics*, 1993.
- McDonald, Torrilhon, *Affordable robust moment closures for CFD based on the maximum-entropy hierarchy*, J. Comp. Phys, 2013.
- Junk, *Domain of Definition of Levermore's Five-Moment System*, J. Stat. Phys, 1998.

# *14-moment maximum-entropy VDF: application to electrons in ExB fields (2 of 2)*

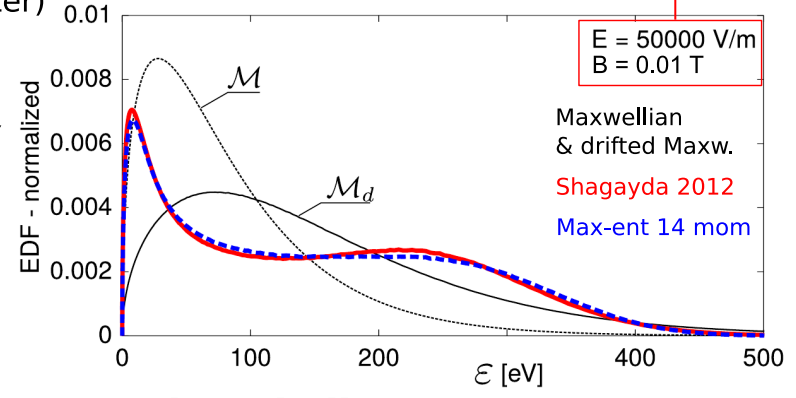
EVDF: $f_{14} = \exp(a + a_i v_i + a_{ij} v_i v_j + a_{i3} v^2 v_i + a_4 v^4)$

Test case: see [Shagayda].
Max-entropy: [Boccelli et al.]

***Good accuracy*** for the ⊥ EVDF (Beta: inverse Hall parameter)




***High accuracy*** for the EEDF




## *Present research & challenges:*

- Characterize waves & instabilities
- Develop chemical terms that include non-equilibrium effects (non-Maxwellian EDF)
- Simulate a coupled ions+electrons system.

## References:


- Shagayda, *Stationary electron velocity distribution function in crossed electric and magnetic fields with collisions*, PoP, 2012.
- Boccelli et al, *A 14-moment maximum-entropy description of electrons in crossed electric and magnetic fields*, PoP, 2020.