Data-driven analysis of breathing and ion-transit modes in 2D hybrid Hall thruster simulations

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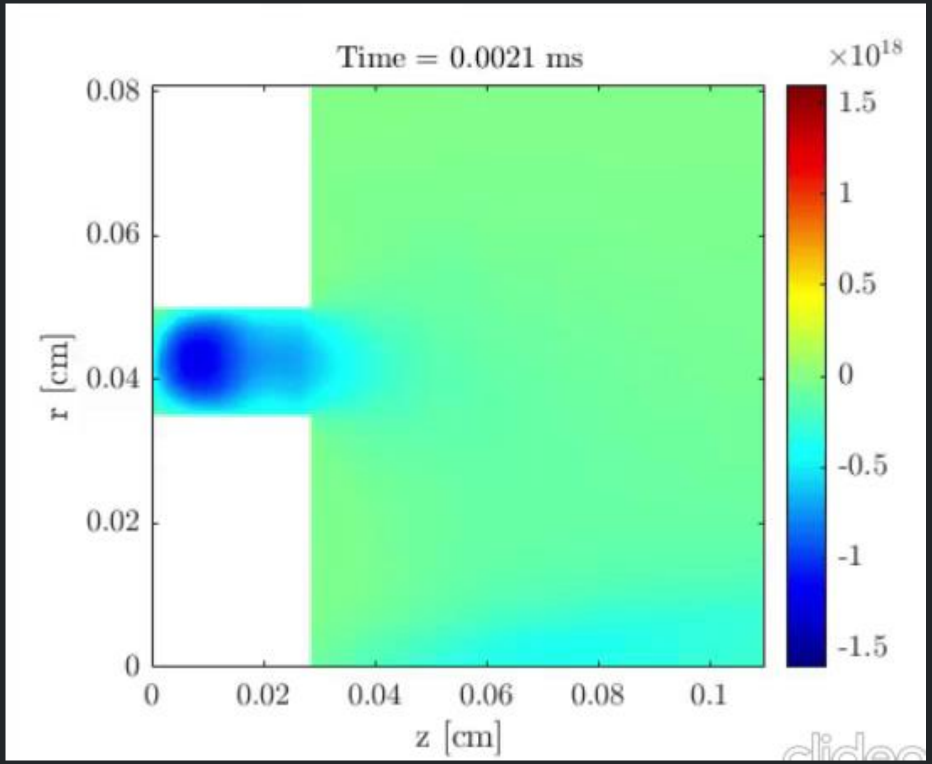
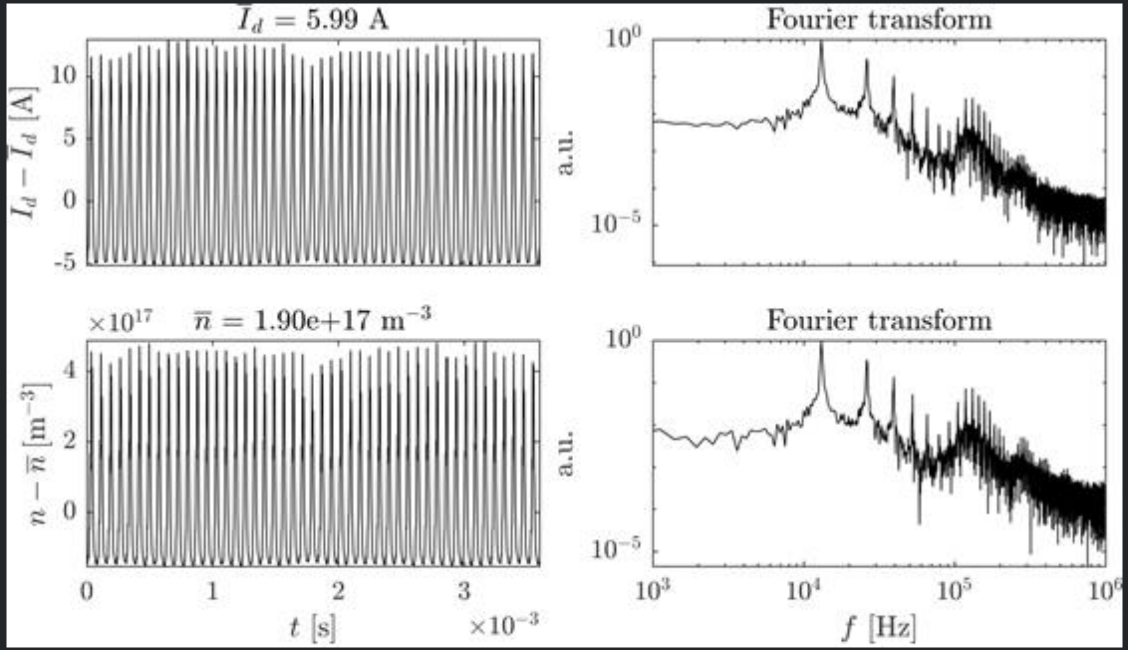
ExB Plasmas Workshop 2022

Madrid, online event

SPT-100 2D hybrid simulations datasets

- 2D axisymmetric hybrid PIC/fluid simulations of a SPT-100 class HET have been produced by Adrián Domínguez-Vázquez (EP2)
  - Total timespan of 3.6 ms, 2400 snapshots
  - Two different operating conditions





Plasma density [m^{-3}] Time average removed: $X - \bar{X}$ Oscillations on the centreline

Higher Order Dynamic Mode Decomposition

- Classic DMD aims to decompose the snapshots as

Amplitude a_k: dynamical relevance

Growth rate δ_k: mode stability/instability

Frequency ω_k: temporal oscillation characterization

$$\boldsymbol{q}_n \approx \boldsymbol{q}_n^{\mathrm{DMD}} = \sum_{k=1}^{K} a_k \psi_k e^{(\delta_k + i\omega_k)t_n}$$

Mode ψ_k: spatial mode shape (complex)

  - Modes are rather ranked by dynamical importance

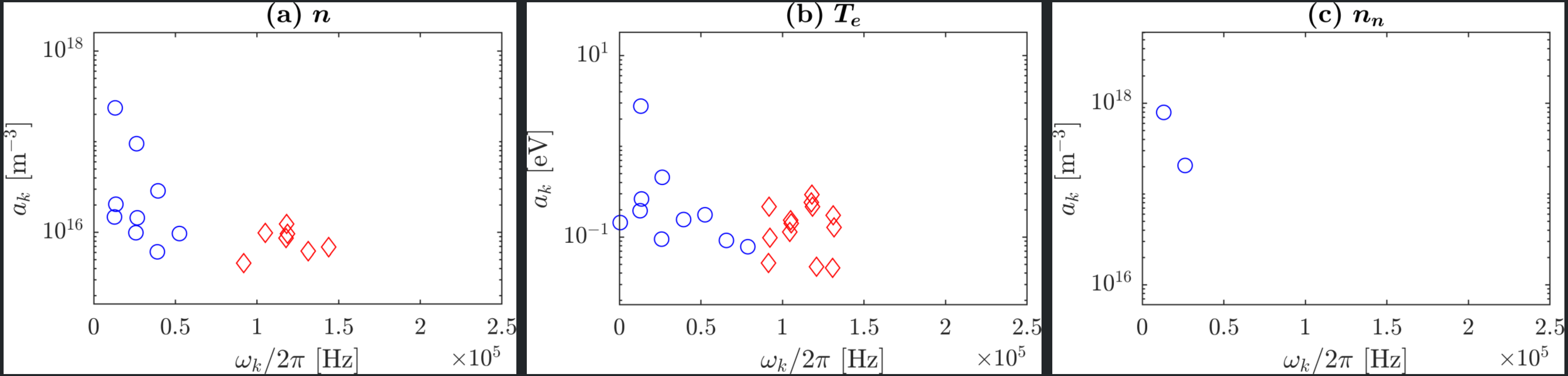
- Standard DMD may produce spurious results for strongly non-linear dynamics with high spectral complexity
  - HODMD allows for improved performances by using time-lagged snapshots

$$\boldsymbol{q}_{n+d} = \boldsymbol{A}_1 \boldsymbol{q}_n + \boldsymbol{A}_2 \boldsymbol{q}_{n+1} + \ldots + \boldsymbol{q}_d \boldsymbol{v}_{n+d-1} \quad \text{for } n = 1, 2, \ldots, N-d$$

- Preliminary noise filtering and iterations are performed for convergence

HODMD diagrams

Nominal case: $V_d = 300\ \mathrm{V}, \dot{m}_A = 5\ \mathrm{mgs}^{-1}$

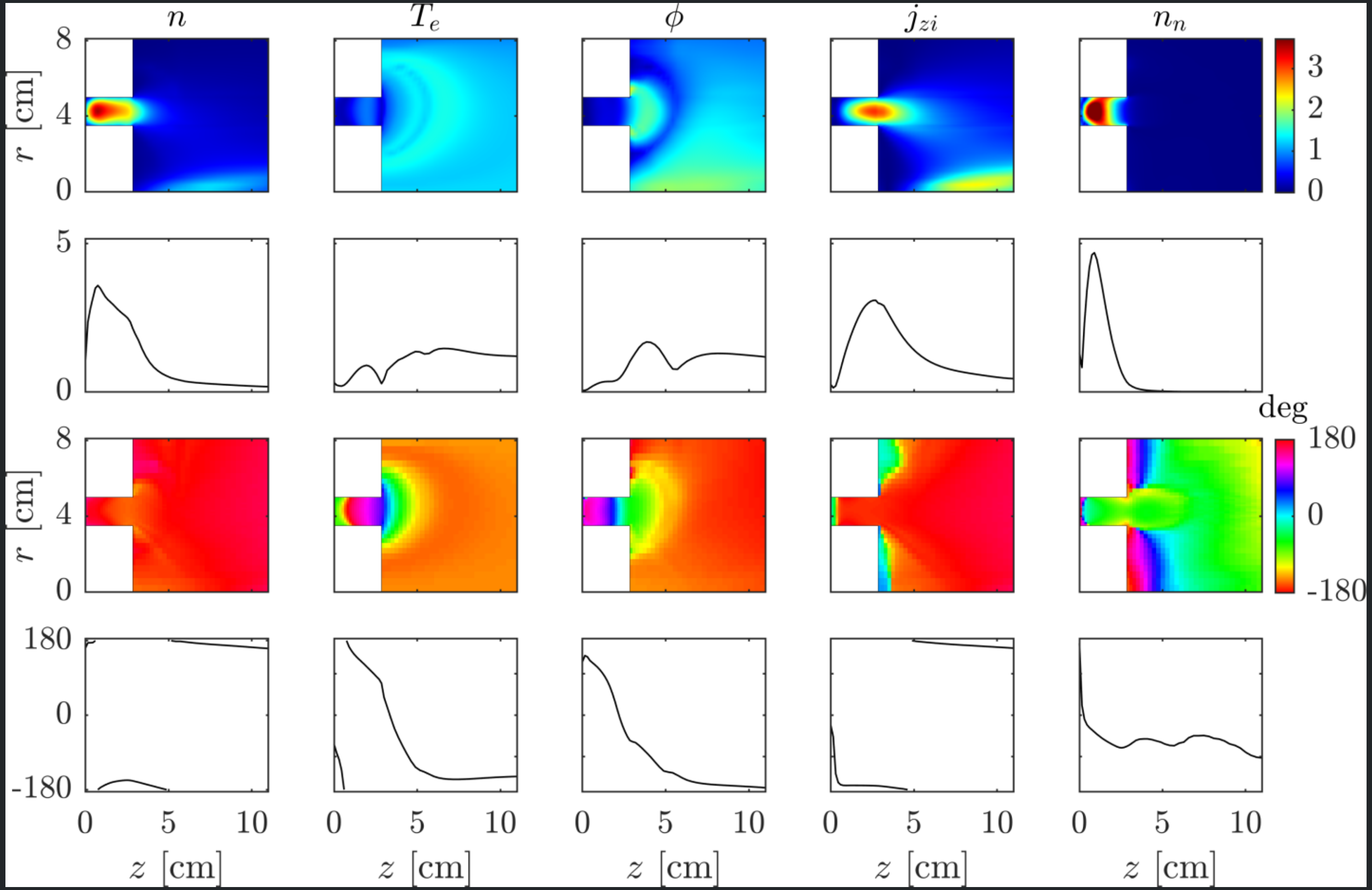


- The two colors represent two different cluster of oscillations : *breathing oscillation* and *ion transit time (ITT) oscillation*
  - Several *replica modes* appear, suggesting highly spectrally complex oscillations

- All variables except neutral density show the same behavior
  - Neutral density does not participate to the ITT oscillation

HODMD dominant breathing mode

- Are the two mentioned cluster of modes really belonging to two different oscillations?

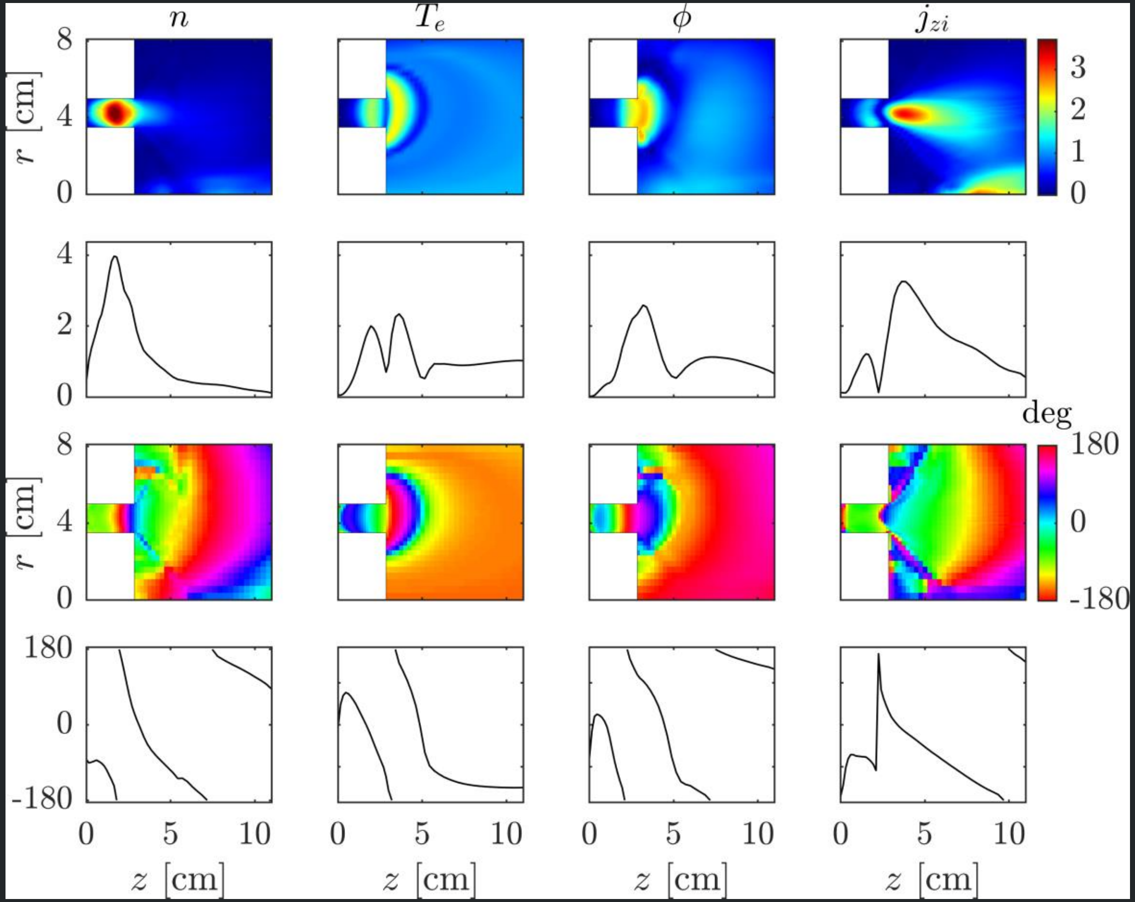


$f_{br} = 13.1$ kHz

- n, j_{zi} and n_n show a global oscillation behavior
  - n_n shows a 90 deg phase shift, in agreement with the predator-prey model
  - Grossly located within the chamber, where the ionization takes place
- ϕ and T_e show a progressive wave behavior (i.e. traveling wave)
  - The progressive structure stops at the cathode, where the phase becomes essentially constant
  - Located in the proximity of the chamber exit, in the acceleration zone

HODMD dominant ITT mode

- Are the two mentioned modes really belonging to two different oscillations?



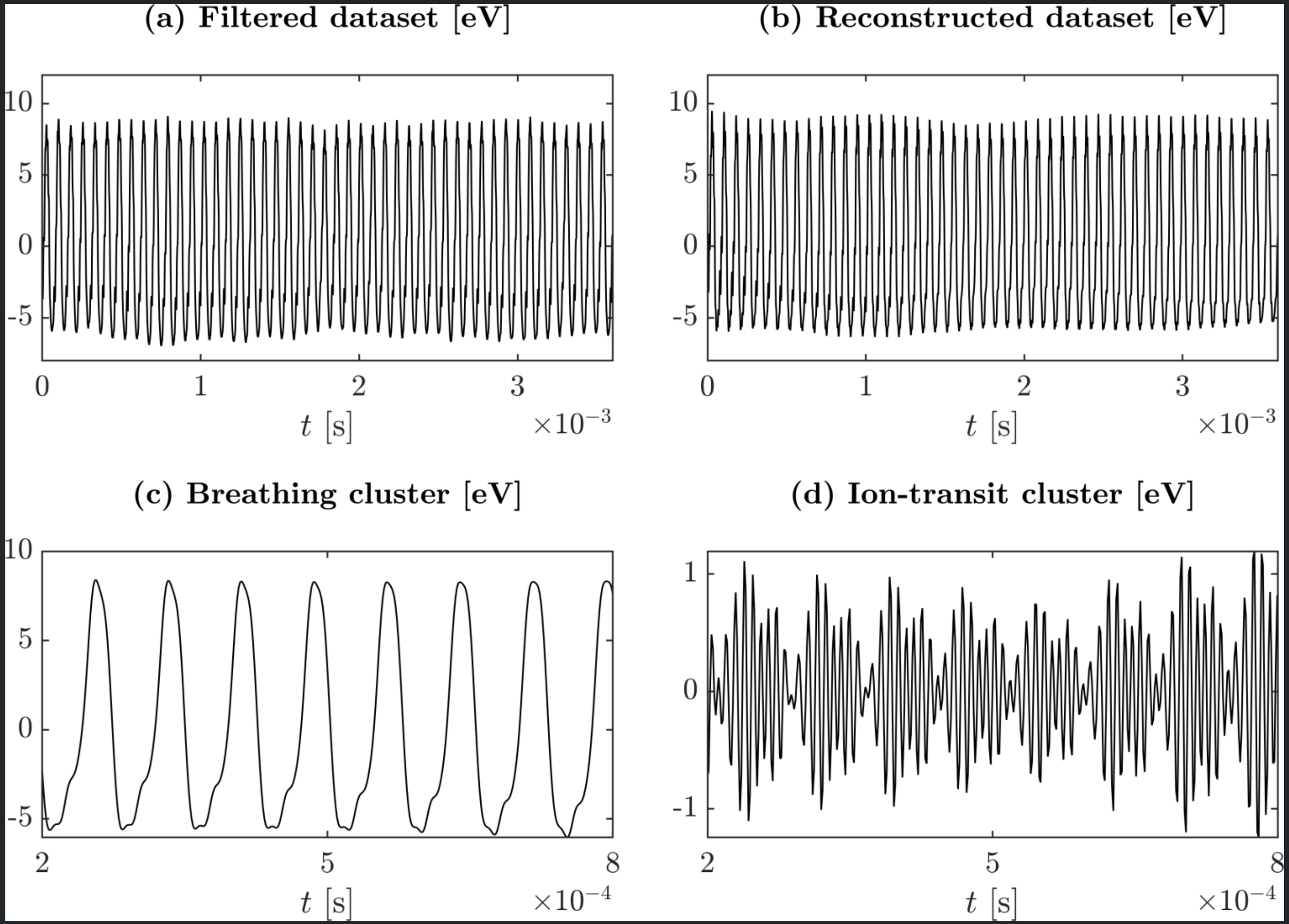
$f_{itt} = 118$ kHz

- All wave structures are now progressive
  - n and j_{zi} switched from global to progressive
- Ion current density shows two magnitude peaks
  - The first one is linked to the plasma density oscillations in the chamber
  - The second one is linked to the ion velocity oscillations induced by the plasma potential
- The ion velocity in the acceleration region divided by the length of such region provides good agreement

$$f_{itt,2} = 127 \text{ kHz}$$

- The phase velocity of n and j_{zi} corresponds with the ion axial velocity

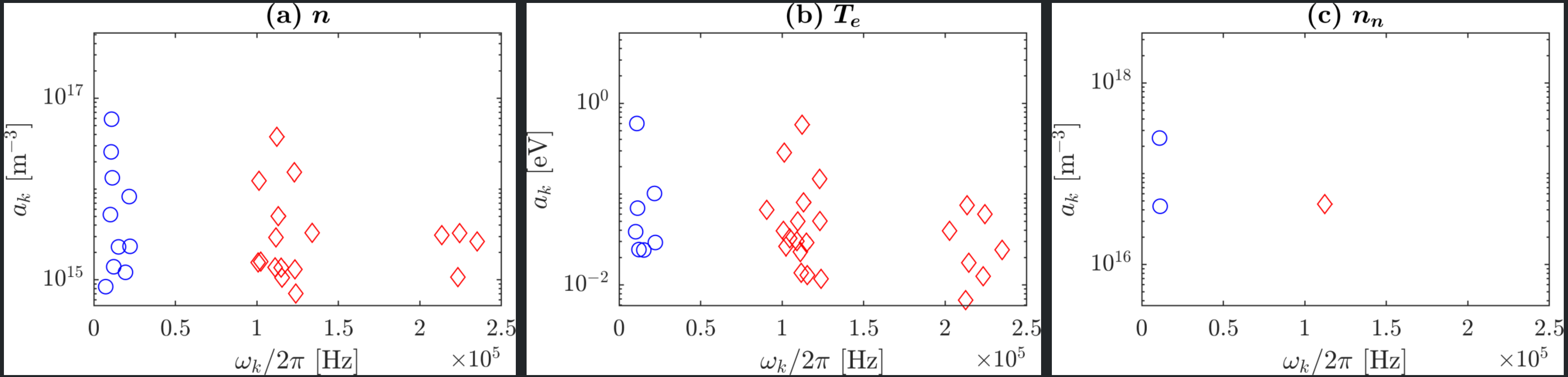
HODMD reconstructions



- The two different clusters can be reconstructed singularly
  - Growth rates are forced to zero due to the reconstruction being asymptotic
- ITT is fully modulated by the breathing dynamics
  - The ITT frequency is an exact multiple of the breathing one
  - The ITT peaks are in phase with the breathing peaks
    - This does not happen for all the cases analyzed

HODMD diagrams

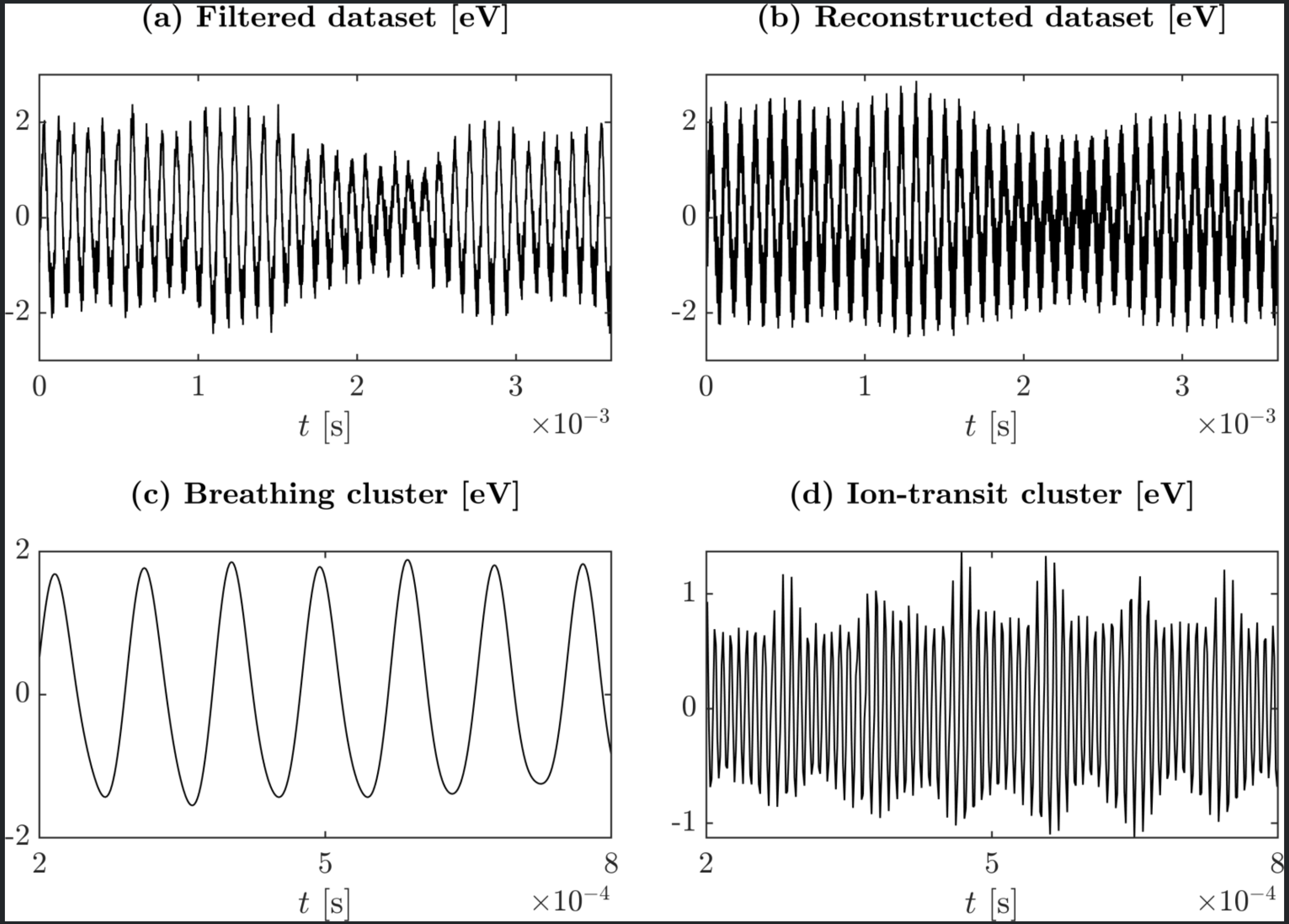
Low voltage case: $V_d = 200$ V, $\dot{m}_A = 5$ mgs^{-1}



- The dynamic importance of breathing and ITT clusters is now comparable
  - The neutral density now shows one mode in the ITT cluster, but with significant lower amplitude

- The number of replica modes is again significant---and has increased
- One new cluster centered around the first harmonic of the ITT dominant mode is now recovered

HODMD reconstructions



- The breathing cluster has a magnitude which is comparable with the one of the ITT
- The modulation of the ITT by means of the breathing dynamic gets weaker
  - The ITT frequency (112 kHz) is no longer an exact multiple of the breathing one (10.1 kHz)
  - The ITT peaks are now out of phase with respect to the breathing ones

Conclusions

- HODMD routine has been applied on simulation data of an SPT-100 HET
  - Breathing and ITT dynamics have been recognized as two different groups of modes with different characteristics
  - The two dynamics have been characterized for different operational regimes

- Results show
  - The breathing mode is one order of magnitude stronger than ITT for the nominal case
    - There is a major modulation of the first one over the second one and the two dynamics are forced in phase
  - Separation of the breathing and ITT mode is supported by a different spatial behavior
  - The spatial location of the oscillation is dependent both on the variable and the dynamics
    - Results agree well with the predator-prey model of the breathing mode and existing ITT models
  - Lowering the voltage implies a significant loss of dominance of the breathing mode
    - The breathing modulation of the ITT dynamics is way less significant

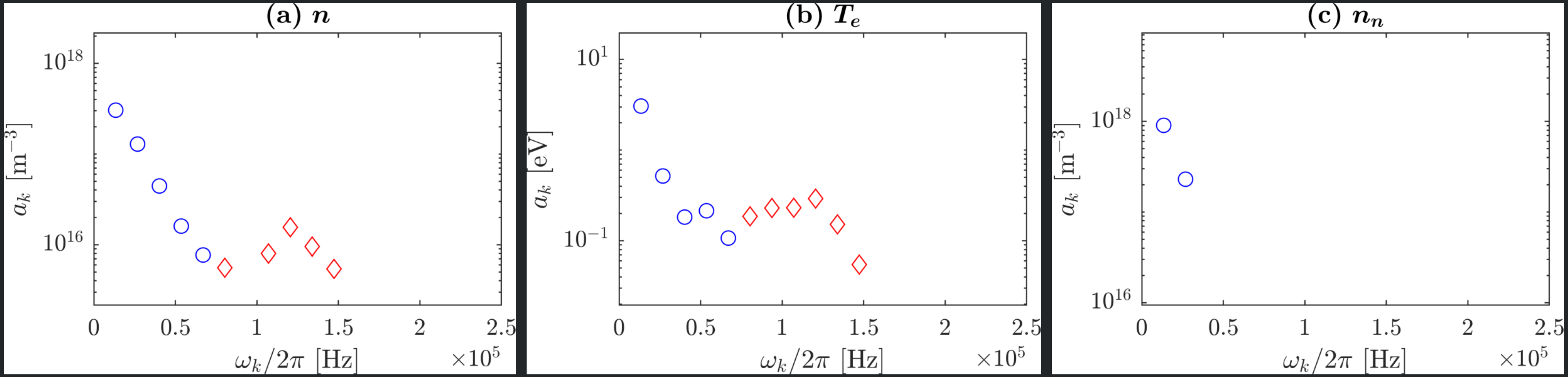
Acknowledgments

This work has been supported by the MARETERRA project, co-funded by the Comunidad de Madrid and the Universidad Carlos III de Madrid, under Grant reference MARETERRA-CM-UC3M.

Backup slides

HODMD diagrams

High flow rate case: $V_d = 300\ \mathrm{V}, \dot{m}_A = 6\ \mathrm{mgs}^{-1}$



- The general behavior is essetially the same as the nominal case
  - One order magnitude of difference between breathign and ITT modes
  - The neutral density shows no modes in the ITT range

- The replica modes now totally disappear, suggesting a spectrally neater behavior

Proper Orthogonal Decomposition

- The data is organized in a set of time-dependent snapshots as

$$q_n = q(t_n) \quad \text{for } n = 1, \ldots, N \quad \longrightarrow \quad Q = [q_1, q_2, \cdots, q_N]$$

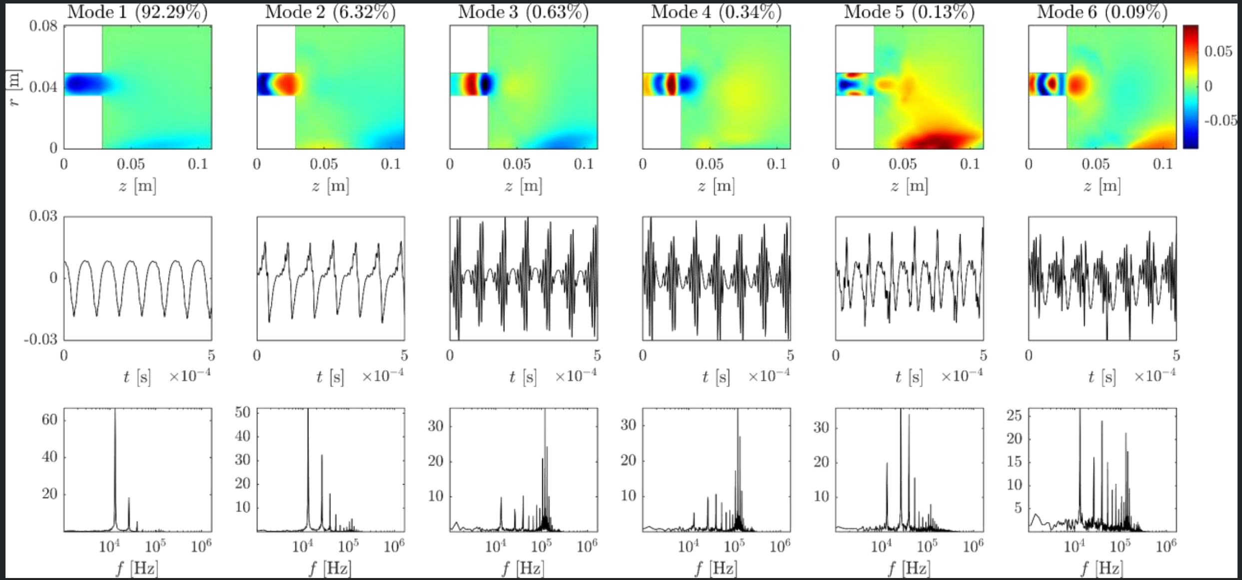
- Standard POD aims to decompose said snapshots as

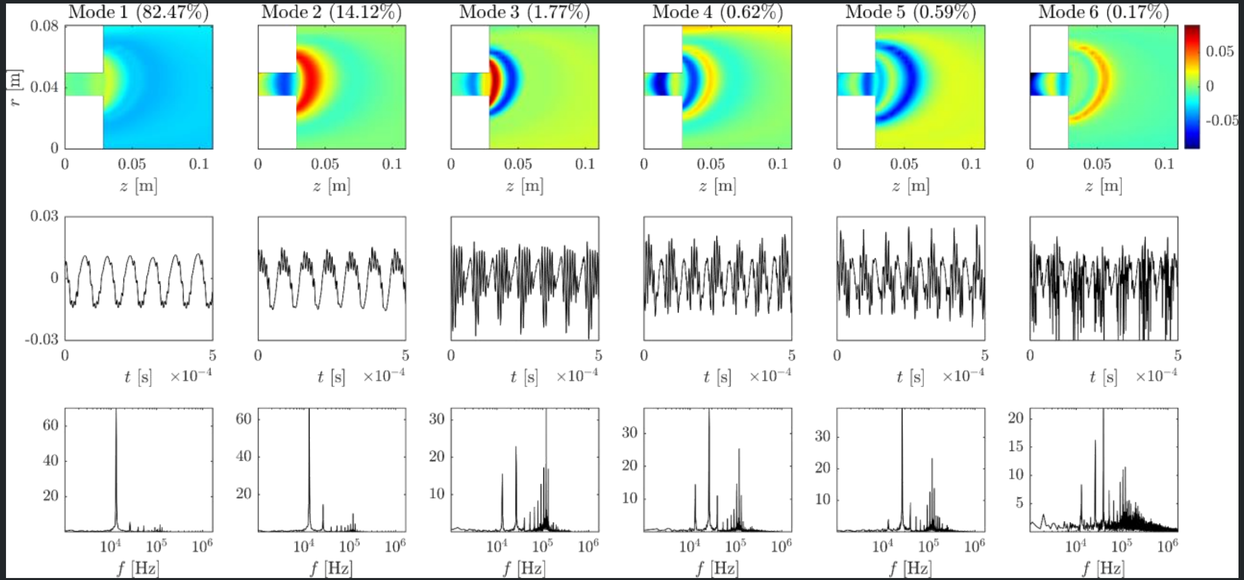
$$q_n \simeq \sum_{k=1}^{K} \alpha_n^k w_k$$

- POD ranks its modes by energetic importance (Singular Values)
  - Each spatial mode is associated to a full-time history and therefore with a full frequency spectrum
    - Different dynamics often tend to mix up
  - Strict isolation of different dynamics is prevented

POD modes and spectra

- POD results have been used as initial benchmark and for comparison

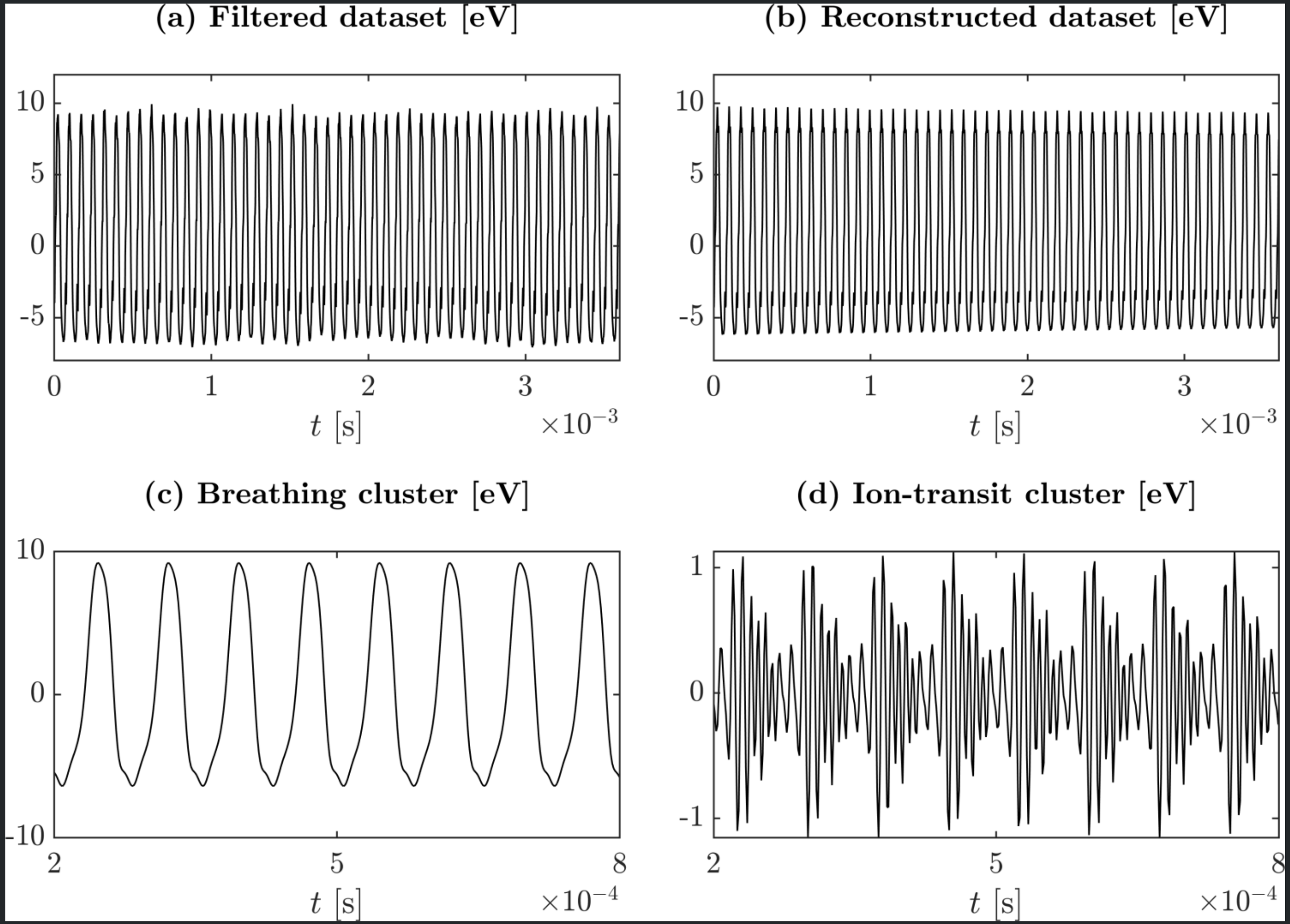




Plasma density $[\mathrm{m}^{-3}]$ $V_d = 300\ \mathrm{V}, \dot{m}_A = 5\ \mathrm{mgs}^{-1}$ **Electron temperature $[\mathrm{eV}]$**

- The first two modes collect up to the 95% of the total energy for both variables
  - They are predominantly dominated by the breathing frequency and its harmonics, but some small peaks in the ITT range appear
- Peaks belonging to the breathing cluster and the ITT one appear within the same POD mode

HODMD reconstructions



- The behavior of the high voltage case essentially resembles the nominal one
- The magnitude of the breathing oscillation is again one order of magnitude stronger than the ITT one
- The modulation of the breathing over the ITT mode is dominant once more
    - The ITT frequency (121 kHz) is an exact multiple of the breathing one (13.4 kHz)
    - The ITT peaks are in phase with the breathing peaks

HODMD dominant breathing mode

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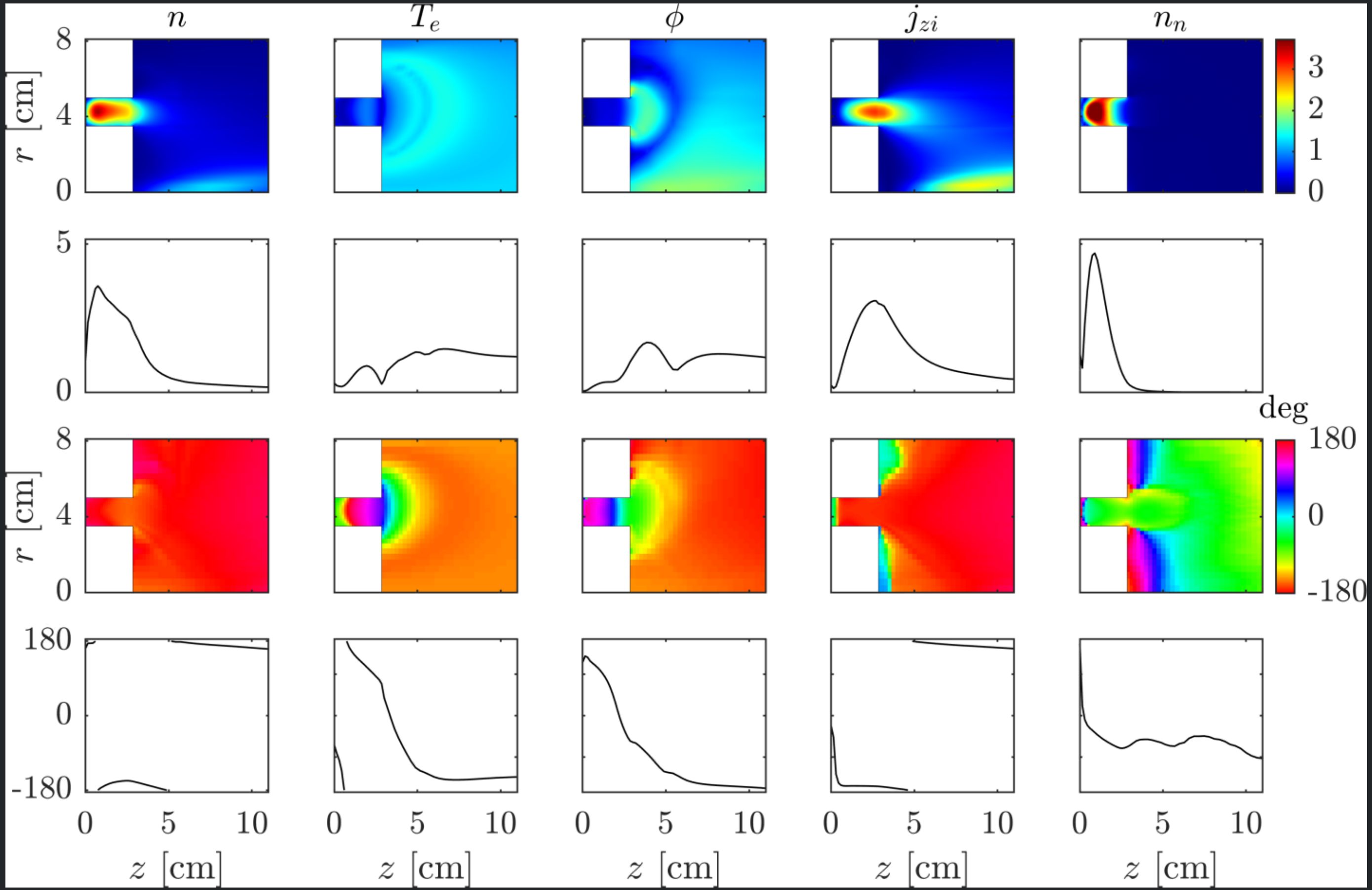

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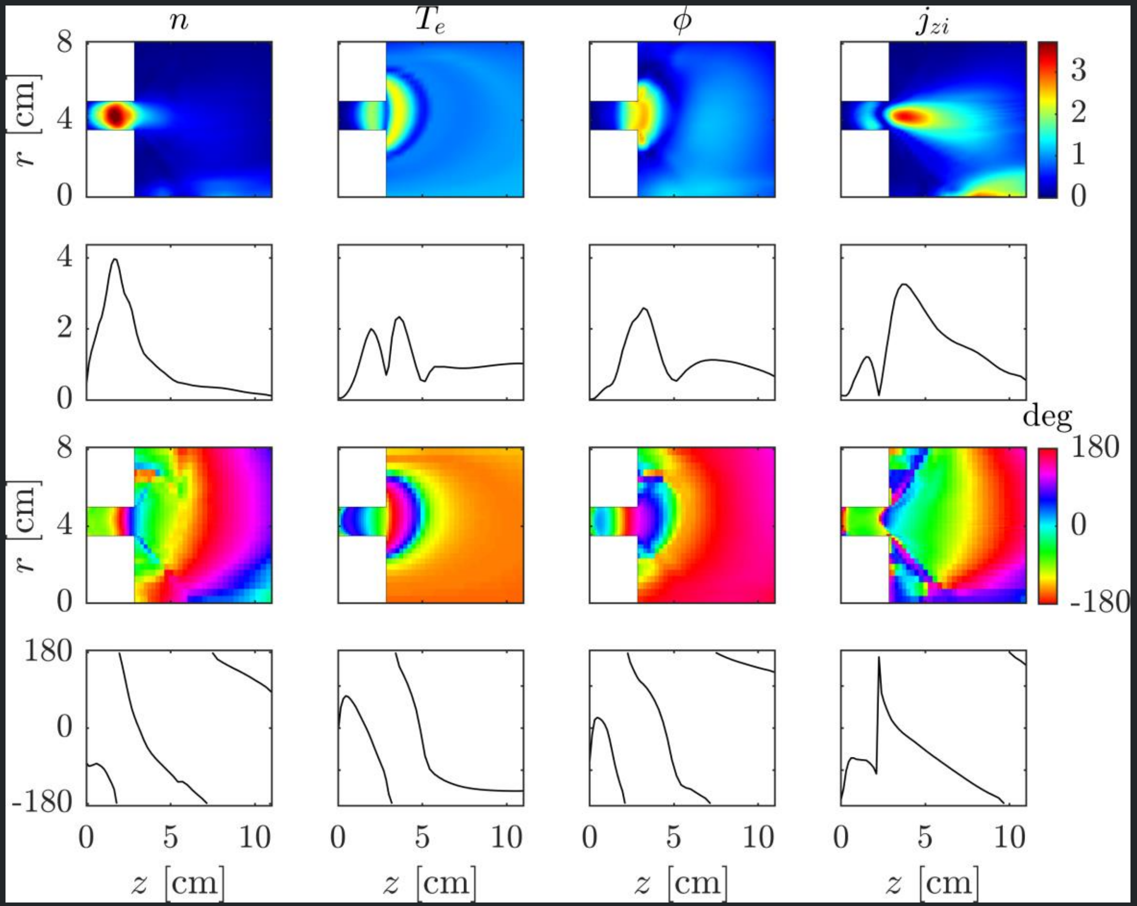

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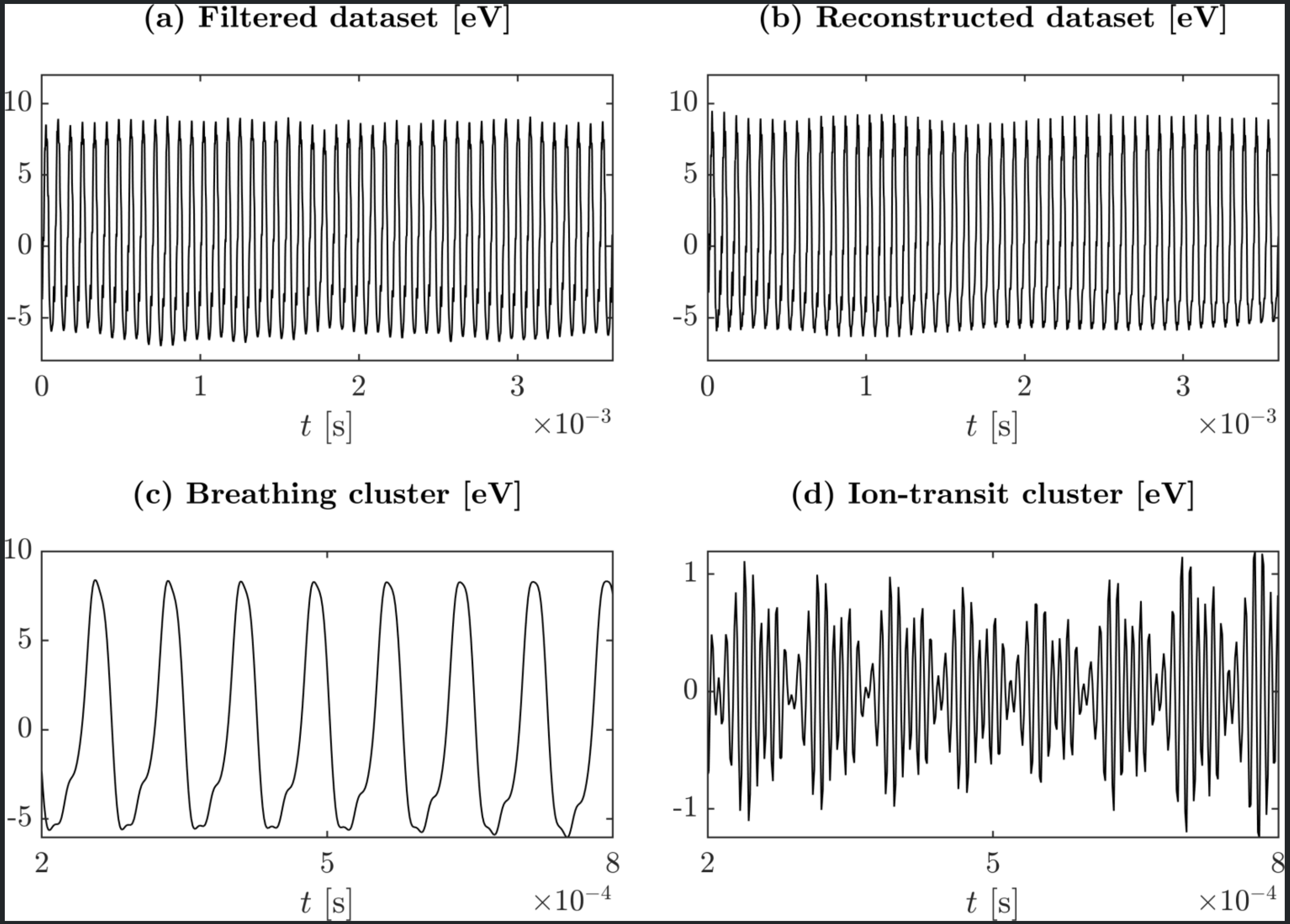

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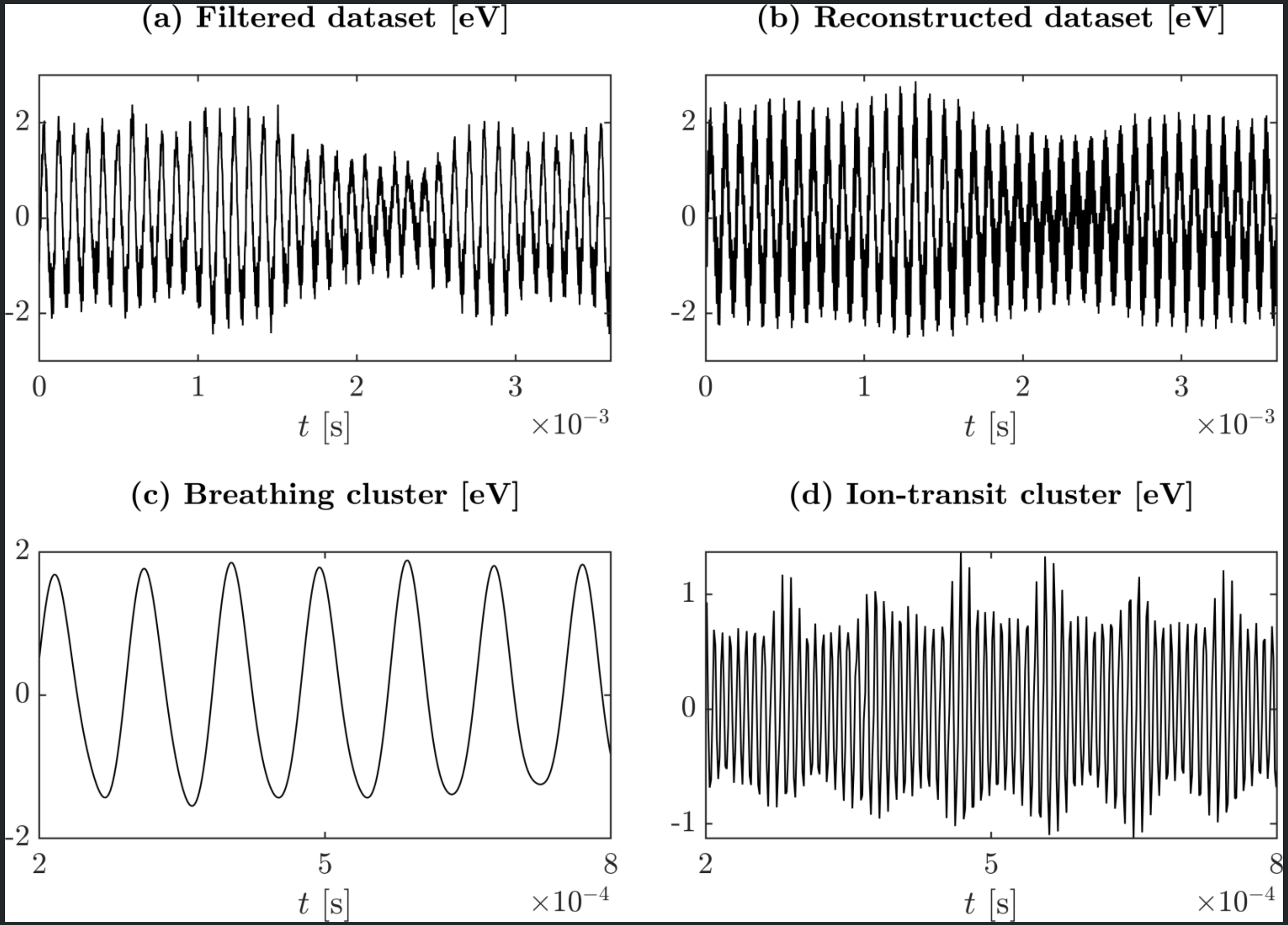

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