Calibrating models of anomalous transport using bi-fidelity surrogates

Thomas A. Marks and Benjamin A. Jorns
University of Michigan, Ann Arbor, Michigan, USA

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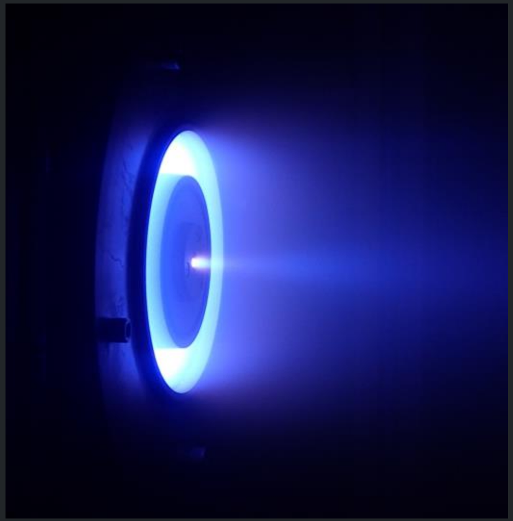
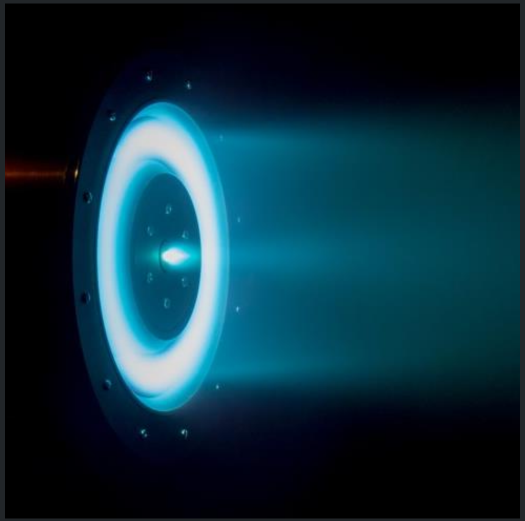
Anomalous electron transport in Hall thrusters

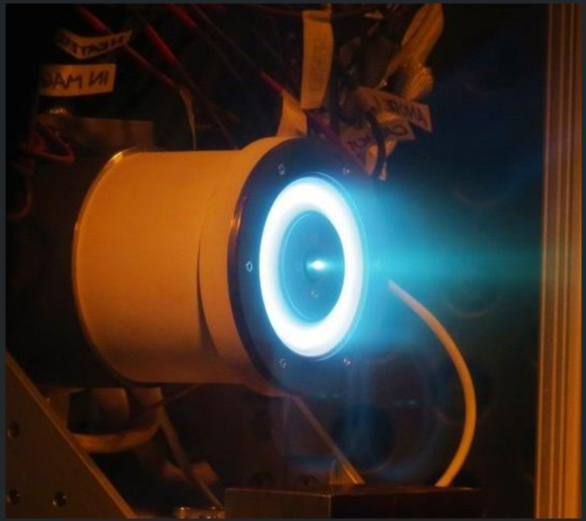
- Hall thrusters are an important EP technology
- Modeling is important to understand their operation and design new thrusters



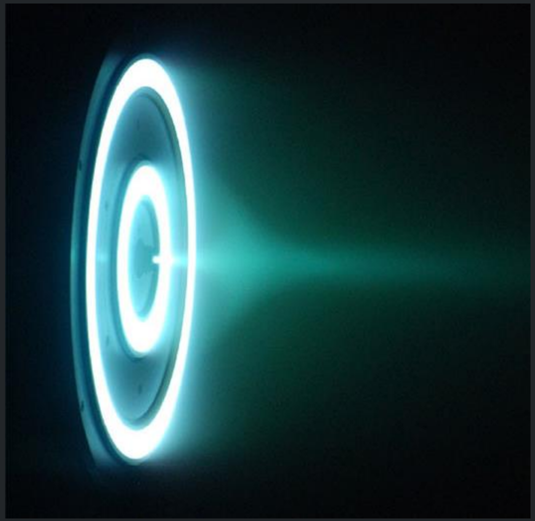
Alternative propellants



Longer lifetimes



Low power



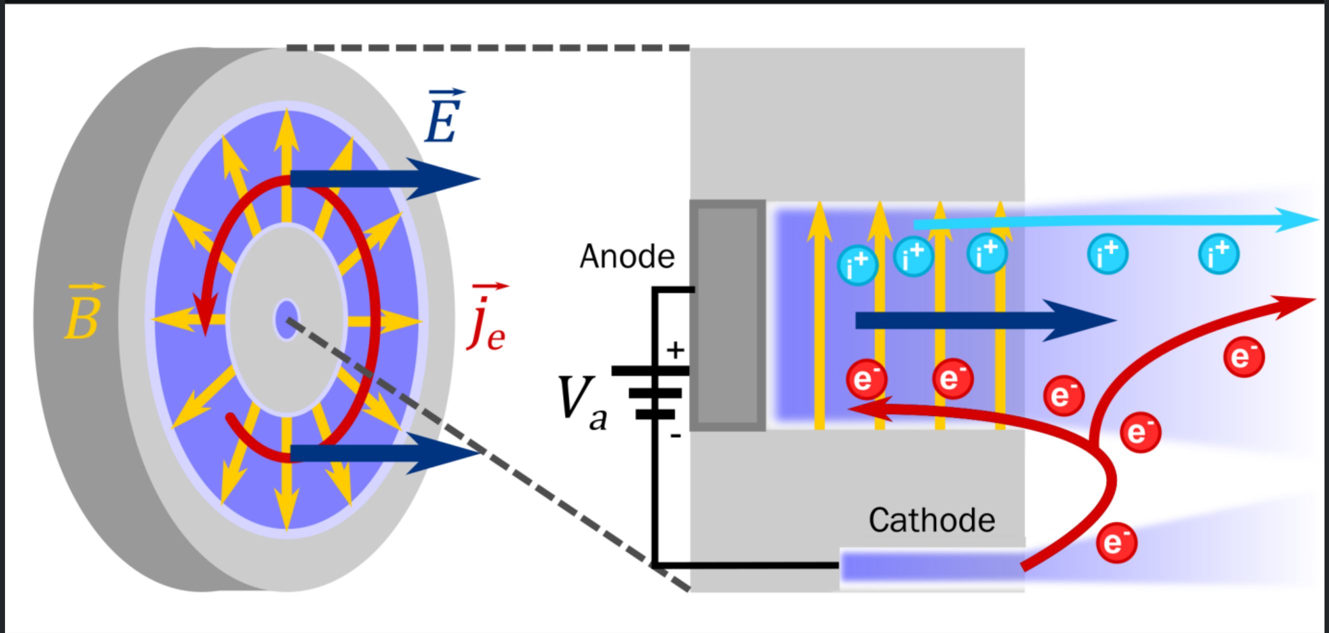
High power

Current simulations not predictive.
Why?

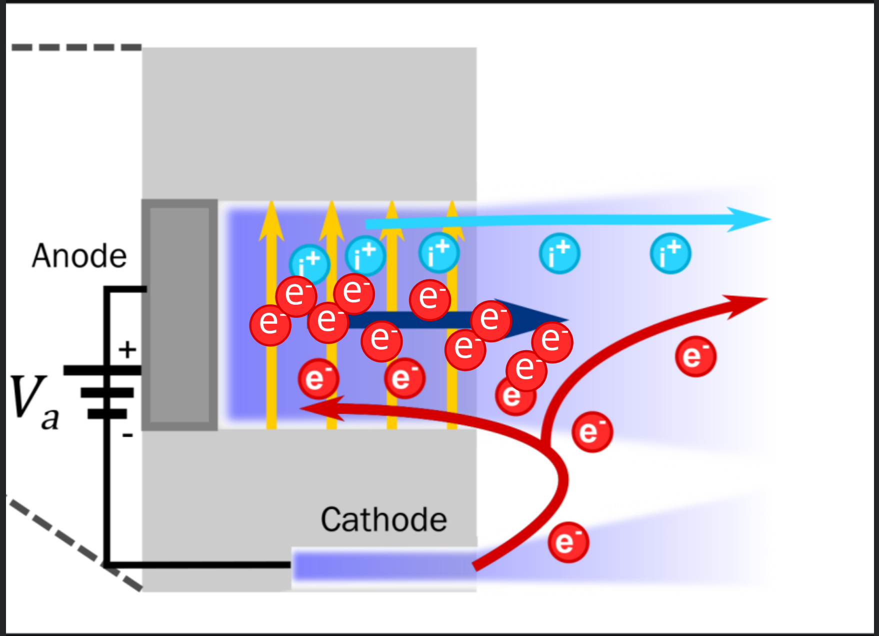
Anomalous electron transport in Hall thrusters

- Electrons diffuse across magnetic field lines far more than classical collisions can account for

Classical collisions



"Anomalous" collisions



$j_{e\perp} \propto \nu_e$

$j_{e\perp} \propto (\nu_e + \nu_{AN})$

Many models proposed, but so far none have been widely employed

Goal: Find a model which works across thrusters with minimal tuning required

Modeling anomalous transport

- Given some model of the anomalous collision frequency:

$$\nu_{AN} = f(p \mid \Theta)$$

Fit coefficients (scalars)

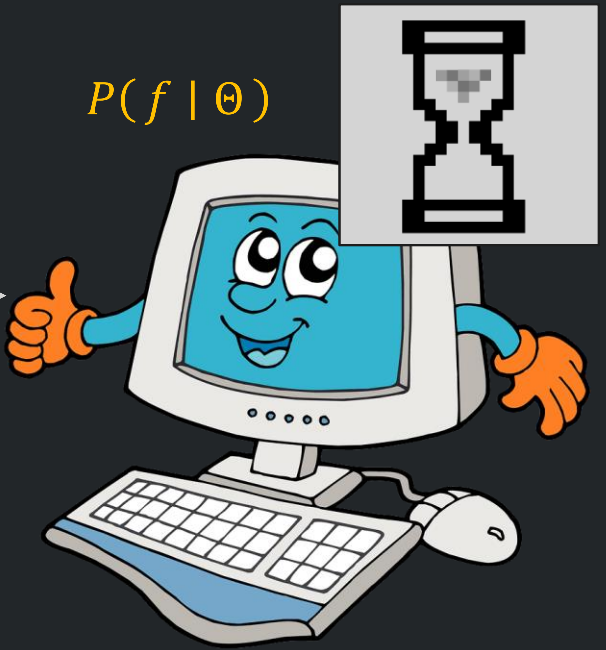
Local plasma parameters

- Denote performance model as $P(f \mid \Theta)$ (aka high-fidelity Hall thruster simulation)
- Motivation: want a probability distribution of likely model coefficients for a given model
- Problem: Given model f and coefficients Θ, evaluating P requires us to run an expensive Hall thruster simulation.

- Model
- Coefficients

→

$P(f \mid \Theta)$



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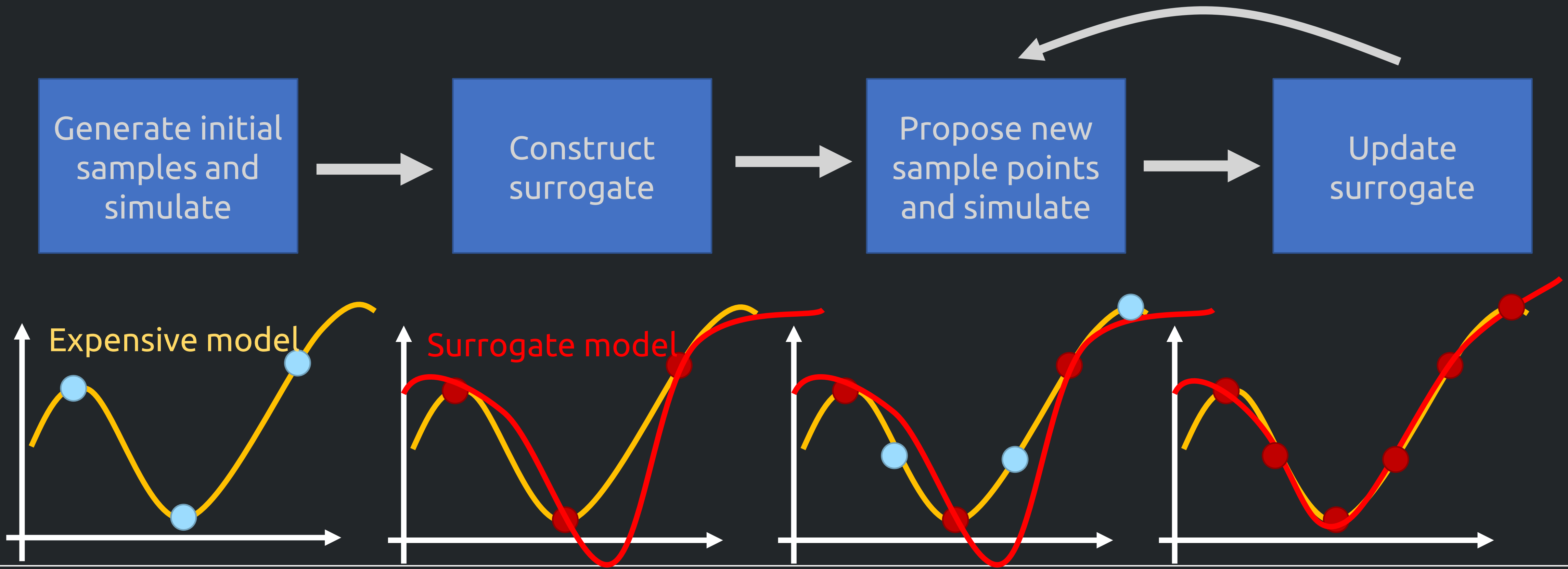
- Thrust
- Discharge current
- Anode efficiency
- Ion velocity trace

…

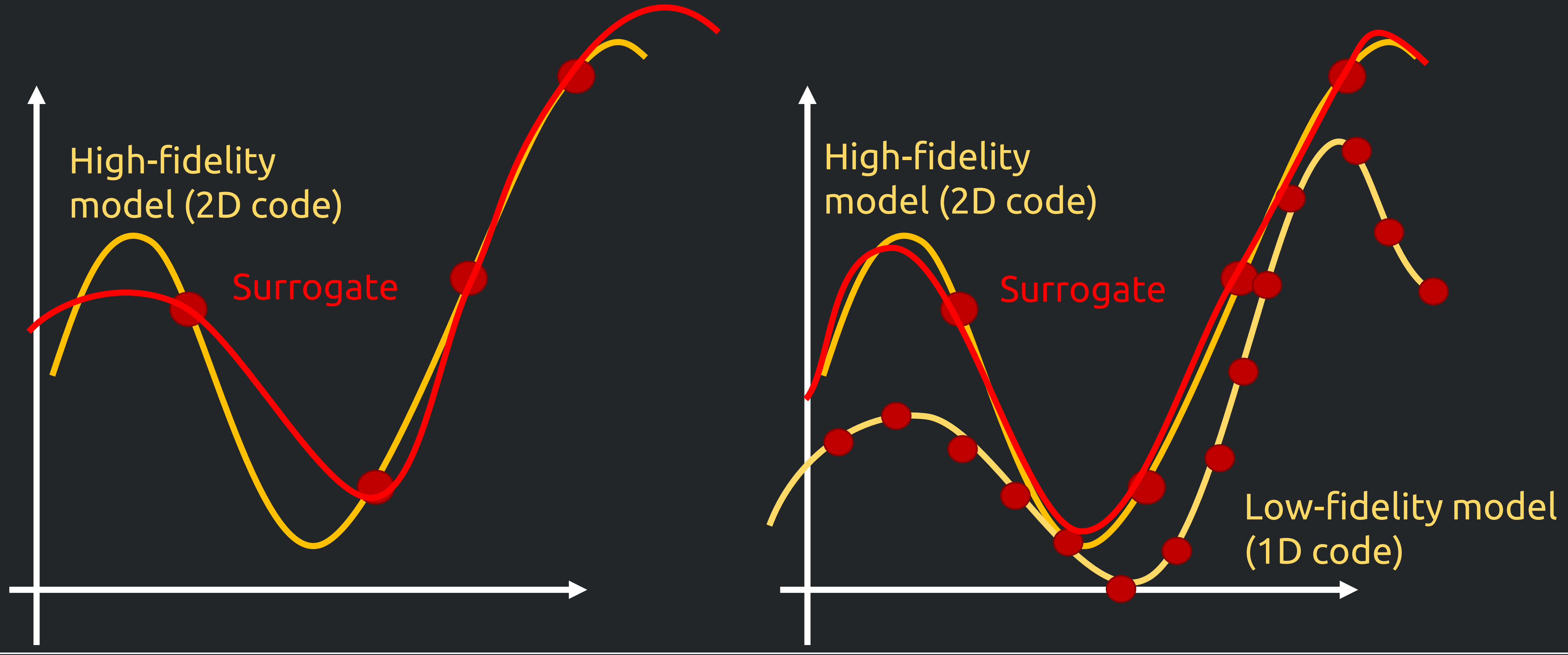
Surrogate modeling

- Hall thruster codes expensive to run, but needed to accurately understand how proposed models affect performance
- Use surrogate (fit function) to model response of code at points not yet tried
- Intelligently select points to simulate to update surrogate
- Once surrogate converged, run parameter inference (MCMC or similar) on surrogate to obtain coefficient distribution



Bi-fidelity surrogates

- Running 2D Hall thruster simulation is expensive, 1D simulations cheap and predict major features
- Can speed up model calibration with a ***multi-fidelity surrogate***
- Instead of inferring expensive model directly, use expensive model to correct cheap model
- Better fit for same number of expensive model evaluations



Models to investigate

- Algebraic (zero-equation) models

	Model	Expression for ν_{AN}	Coefficients	Proposed mechanism
[1]	Two-zone Bohm	$\begin{cases} \alpha_0\,\omega_{ce} & z < L \\ \alpha_1\,\omega_{ce} & z \geq L \end{cases}$	$\alpha_0,\ \alpha_1$	Bohm diffusion
[2]	Turbulence I	$\omega_{ce}\sqrt{\frac{\lvert J_{e\perp}E_\perp \rvert}{Ken_e c_e^2 B}}$	K	Turbulent viscosity (energy cascade)
[3]	Turbulence II	$\frac{1}{K}\frac{\lvert\nabla\cdot(\mathbf{u_i}n_eT_e)\rvert}{m_e c_s n_e v_{de}}$	K	Azimuthal instability saturated by ion-wave trapping and wave convection
[4]	Turbulence III	$\frac{1}{K}\omega_{ce}\left(\frac{1}{1+(C\ \nabla v_{de})^\alpha}\right)$	$K,\ C,\ \alpha$	Turbulent transport suppressed by shear stress
[5]	Data-driven	$\omega_{ce}\left(c_0 + \frac{c_1\lvert\mathbf{u_i}\rvert}{c_2 c_s + v_{de}}\right)$	$c_0,\ c_1,\ c_2$	None

- One-equation and two-equation models

$$\frac{\partial W_T}{\partial t} + \vec{u}_i\cdot\nabla W_T = 2W_T\left(\langle\omega_i\rangle - \langle\omega_{loss}\rangle - \nabla\cdot\vec{u}_i\right).$$

$$\frac{\partial\langle\omega_i\rangle}{\partial t} + \vec{u}_i\cdot\nabla\langle\omega_i\rangle = 2\langle\omega_i\rangle\left(c_1 M_e\omega_{pi} - \langle\omega_i\rangle\right) - c_2\frac{1}{M_e}\langle\omega_i\rangle^2\frac{W_T}{n_eT_e}.$$

[6]
$$\nu_{AN} = c_3\sqrt{\frac{m_i}{m_e}}\frac{1}{M_e}\langle\omega_i\rangle\frac{W_T}{n_eT_e},$$

[1] G. J. Hagelaar, J. Bareilles, L. Garrigues, and J. P. Boeuf, "Two-dimensional model of a stationary plasma thruster," Journal of Applied Physics, vol. 91, no. 9, pp. 5592–5598, May 2002. [Online]. Available: https://doi.org/10.1063/1.1465125
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[5] B. Jorns, "Predictive, data-driven model for the anomalous electron collision frequency in a Hall effect thruster, \ "Plasma Sources Science and Technology, vol. 27, no. 10, Oct 2018.
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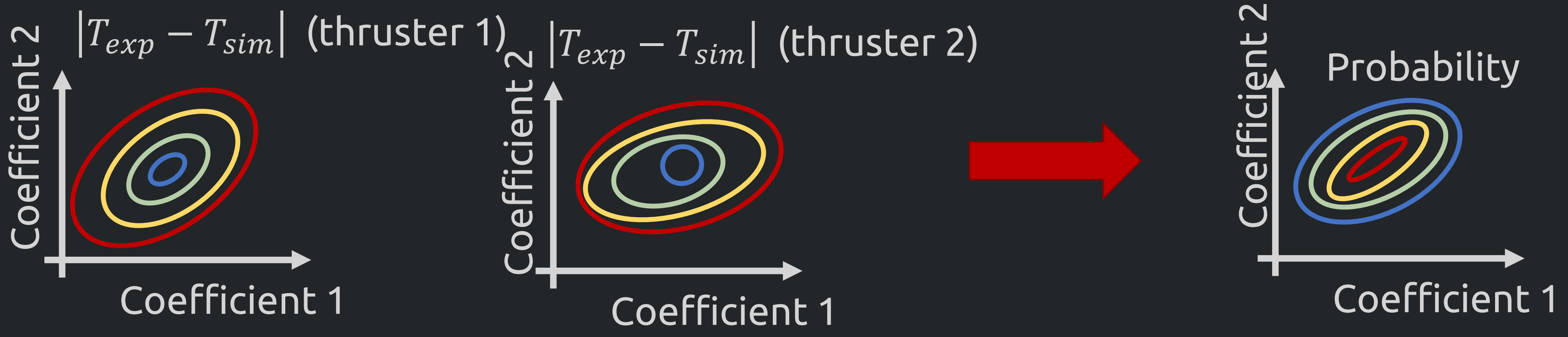
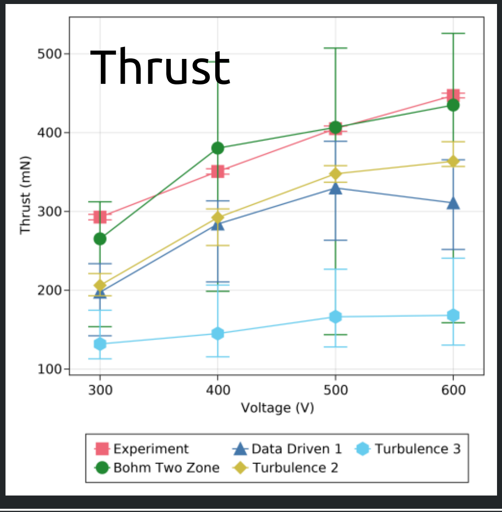
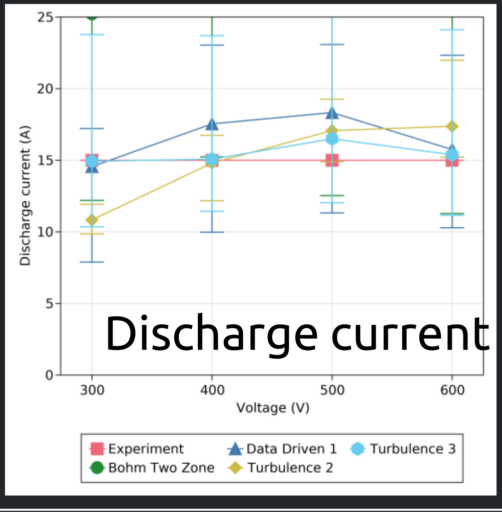
Approach

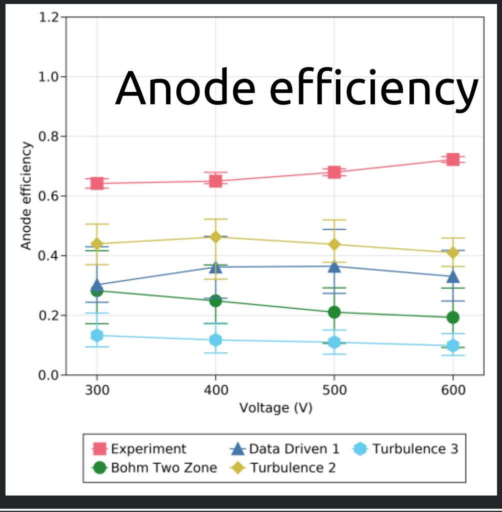
- Construct bi-fidelity surrogate of performance variables for several thrusters and operating conditions
- Use experimental data and surrogate to perform parameter inference (find most likely θ across all cases)

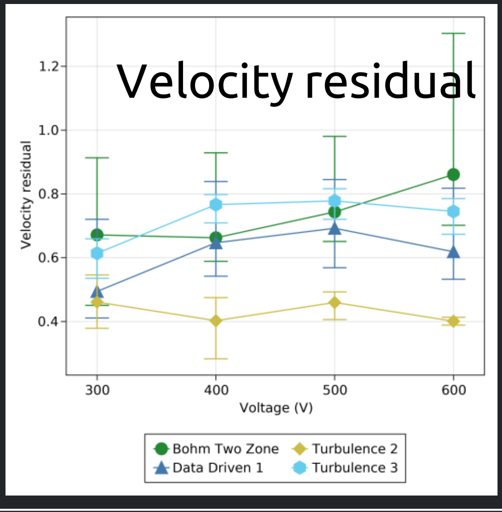


- Sample from coefficient distribution to obtain probabilistic performance predictions (uncertainty quantification)









Acknowledgments

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Contact: marksta@umich.edu