Comparison between 3D Magnetized Plasma Plume Solutions obtained with full FD and hybrid FV-FD schemes

Alberto Modesti
Universidad Carlos III de Madrid

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Contents

- Magnetized plasma plume simulations
  - Plume characteristics
- EP2PLUS in-house code
  - Magnetized electron fluid model
    - Full Finite Differences 2nd order scheme
    - Hybrid FV-FD staggered 1st order scheme
- Numerical simulations
  - Comparison between the two schemes
- Conclusions and future works
  - Application: numerical diffusion assessment

Magnetized plumes and EP2PLUS features

- Plasma plumes from electric thrusters (HET, HEMPT, ECRT, HPT)
  - Weakly collisional
  - Magnetization mainly affects electrons
    - Geomagnetic field always introduces some electron magnetization in LEO

- EP2PLUS 3D hybrid simulator:
  - PIC-fluid (electron fluid to reduce spatial and time resolution)
  - PIC module for heavy species and collisions
  - Magnetized electron fluid model
  - Chamber and free-space scenariosof interest for many applications and developments

EP2PLUS magnetized electron fluid model

- 3D magnetized electron fluid model:

$$0 = -\nabla p_e - en_e(\boldsymbol{E} + \boldsymbol{u_e} \times \boldsymbol{B}) - \sum_{s=1}^{L} \nu_{es} m_e n_e (\boldsymbol{u_e} - \boldsymbol{u_s}) \qquad \nabla \cdot \boldsymbol{j} = 0$$

- Hypotheses
  - Stationary and massless electron fluid
  - Isotropic and polytropic electron closure

Normalized conductivity tensor

$$\bar{\bar{K}} = \begin{bmatrix} 1 & \chi b_z & -\chi b_y \\ -\chi b_z & 1 & \chi b_x \\ \chi b_y & -\chi b_x & 1 \end{bmatrix}^{-1}$$

$$\chi = \frac{\omega_{ce}}{\nu_e} = \frac{eB}{m_e \nu_e}$$

Hall parameter

$$\sigma_e = \frac{e^2 n_e}{m_e \nu_e}$$

Electron conductivity

$$\boldsymbol{j_c} = \frac{en_e}{\nu_e} \sum_{s=1}^{L} \nu_{es} \boldsymbol{u_s}$$

Collisional current density

$$\nabla \Phi = \nabla \phi - \frac{\nabla p_e}{n_e}$$

Thermalized potential gradient: correction due to magnetic field and collisions

Resulting equations:

$$\begin{cases} \boldsymbol{j} = -\bar{\bar{K}} \cdot (\sigma_e \nabla \Phi + \boldsymbol{j_c}) - \boldsymbol{j_i} \\ \nabla \cdot \boldsymbol{j} = 0 \end{cases}$$

Full FD vs hybrid FV-FD

- Elliptic equation in Φ (to be discretized)
  - Substituting Ohm's law into $\nabla \cdot \boldsymbol{j} = 0$

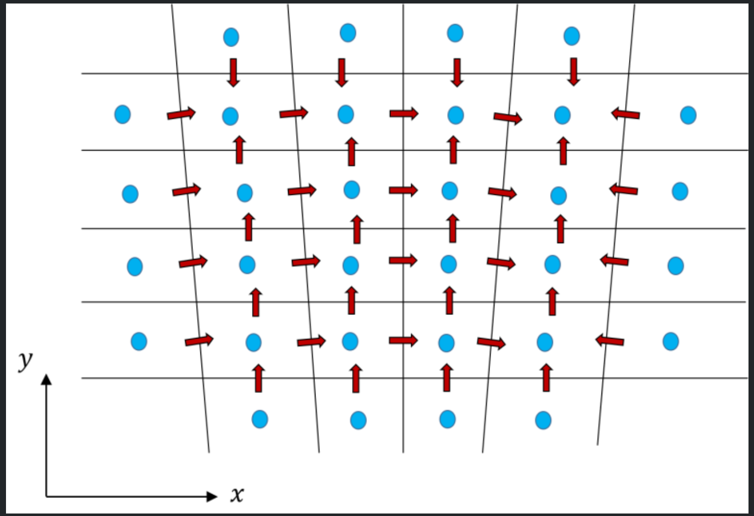
$$\overline{\overline{K}} : \nabla\nabla\Phi + \nabla\Phi \cdot \left(\nabla \cdot \overline{\overline{K}}\right) + \overline{\overline{K}} \cdot \nabla\Phi \cdot \nabla \ln(\sigma_e) = \frac{\nabla \cdot \left(\boldsymbol{j_i} - \overline{\overline{K}} \cdot \boldsymbol{j_c}\right)}{\sigma_e}$$

- BC: local current free condition, strong closure
  - BC on currents translated into a condition on a directional derivative of the thermalized potential
  - 2nd order FW/BW scheme discretization

$$\sigma_e \nabla\Phi \cdot \left(\overline{\overline{K}}^T \cdot \mathbf{1}_n\right) = \left(\boldsymbol{j_i} - \overline{\overline{K}} \cdot \boldsymbol{j_c}\right) \cdot \mathbf{1}_n \qquad (j_n = 0)$$

The direction is between the normal to the boundary and the parallel to B

- Main innovative features:
  - Unknowns in staggered locations
    - Interpolation needed between PIC and fluid properties
  - Hybrid FV-FD approach (continuity and momentum)
    - Conservative scheme
  - Centered FD schemes also at boundaries: lower discretization errors
- First discretize, then substitute ohm into continuity
- Final 1st order system in Φ
- BC as explicit $j_n = 0$ condition



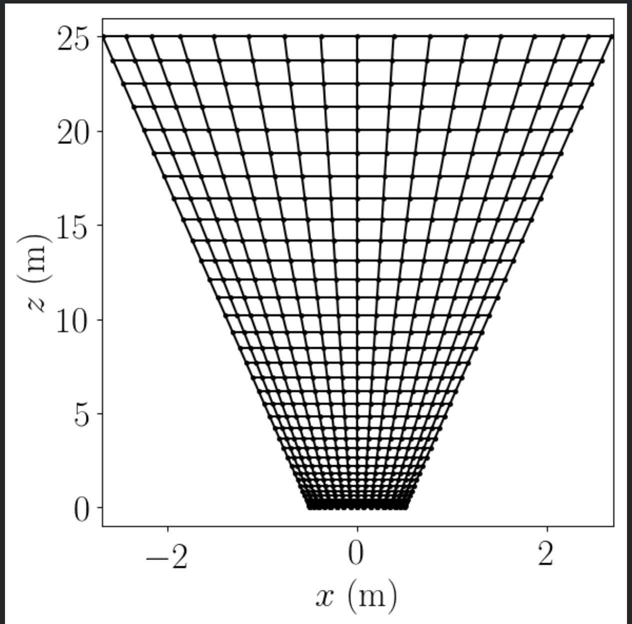
j_n

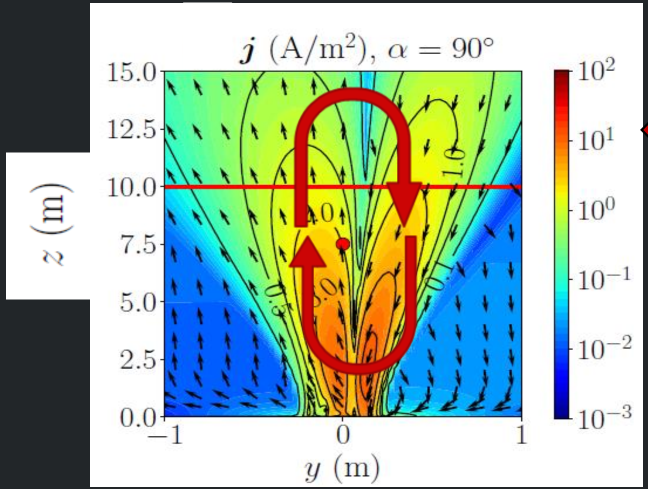
Φ

Schemes comparison: setup and scenario

- Structured mesh setup
  - 81x81x201 nodes mesh parabolical in *z* (linear dz increase)
  - *Z*max = 25m
  - Conical mesh with divergence of 5°
- Physical setup
  - Uniform geomagnetic field = 0.5 Gauss
  - Max Hall parameter = 100
  - α = magnetic field orientation wrt *z*

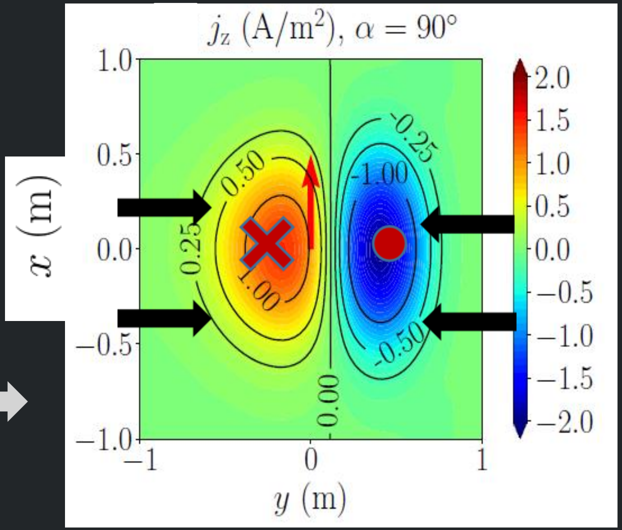
- Scenario: geomagnetic expansion of plume
  - Force on ions due to the gradient of the thermalized potential opposite to the Lorentz force (0 net deflection)





Diamagnetic current loop

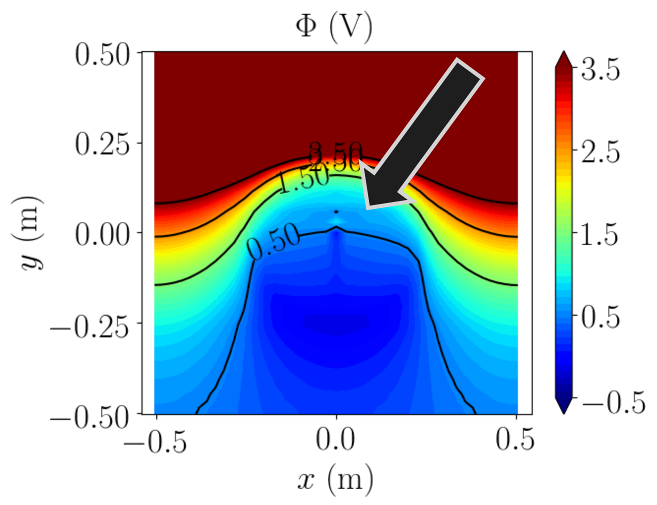
Plume compression due to jxB



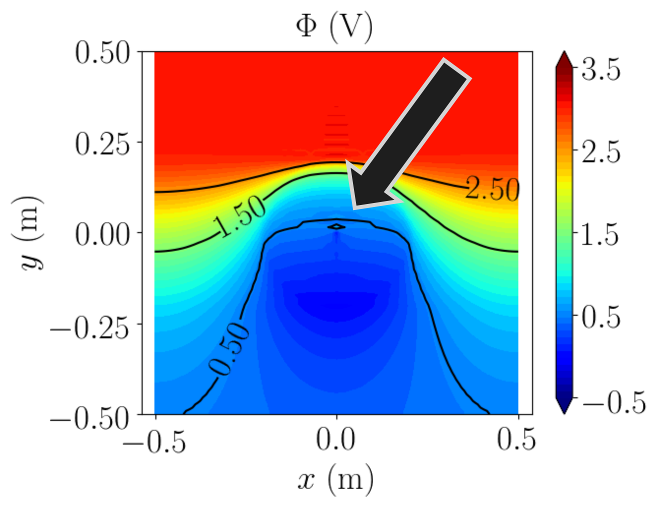
Cichocki, F., Merino, M., and Ahedo, E., "Three-dimensional geomagnetic field effects on a plasma thruster plume expansion," Acta Astronautica, Vol. 175, 2020, pp. 190 – 203.

Schemes comparison: continuity error

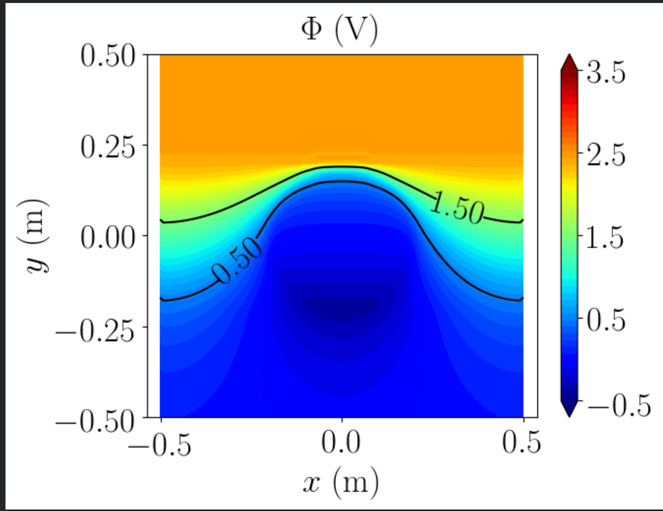
- Dirichlet effect in full FD scheme
  - Node where Dirichlet condition is imposed: Ohm's law is not resolved there
    - FD truncation error appears there, perturbing the surrounding solution
    - Effect may propagate over time
  - Artificial source or sink of electric current (being the scheme not conservative)
    - Artificial current source decreases with approximately the square of the mesh size (here -4.3 A vs -1.2 A with double resolution)
  - Higher resolution is required to reduce its effects, higher simulation time
  - Staggered FD-FV approach is conservative, so no discontinuity at Dirichlet node



2nd order FD
(41x41x101 nodes, max Hall 35)



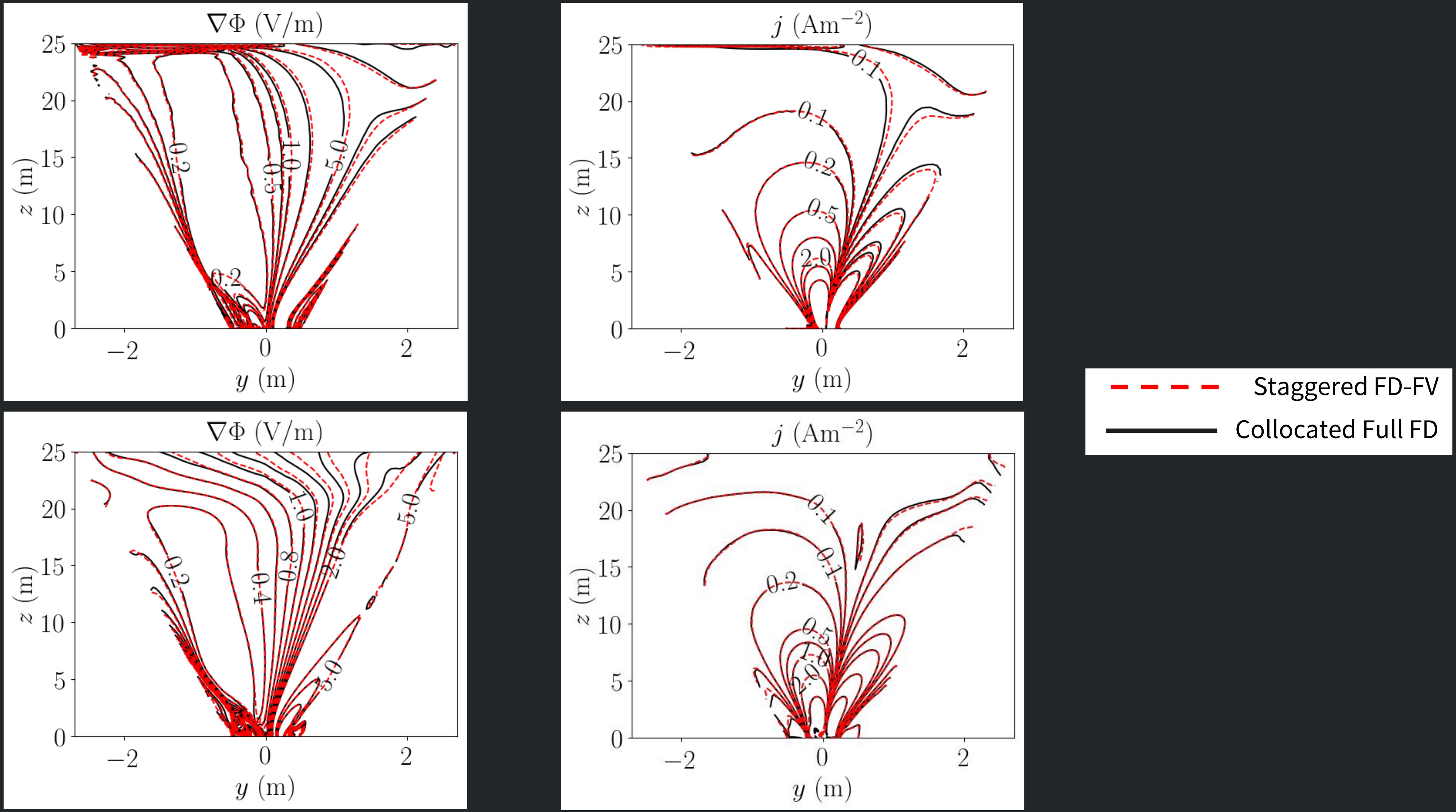
2nd order FD
(81x81x201 nodes , max Hall 35)



1st order FD-FV
(81x81x201 nodes , max Hall 35)

Schemes comparison: solution shape

- α = 90°:
- α = 30°:



Conclusions

- Conclusions
  - Staggered scheme was worth to implement, since it brings advantages in the quality of the solution:
    - Allows to reduce resolution previously required for Dirichlet error, being a conservative scheme
    - Speed up the simulation as a consequence (coarser mesh)
    - With hybrid FV-FD scheme, energy and heat equations discretization can benefit of the same approach
  - Drawbacks:
    - More interpolations needed
    - Unknown locations: nontrivial numbering systems + ghost cells

- Future work
  - Investigate differences at higher Hall parameters
  - Numerical diffusion assessment

Acknowledgments

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- PROMETEO-CM (grant agreement Y2018/NMT-4750)
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email: amodesti@pa.uc3m.es / ep2@uc3m.es
web: ep2.uc3m.es

Backup slides

Backup infos

- Mesh characteristics
  - 2 nodes of spatial smoothing
  - Time averaging: 8000 steps
  - Compare solutions at 1st step to avoid continuity error propagation in time

Reference ion density	$1.36 \cdot 10^{16}\ m^{-3}$
Injected Xe ions flow	2.38 mg/s
Ions injection axial velocity	39 km/s
Background neutral density	$7.16 \cdot 10^{17}\ m^{-3}$
95% ion current radius	0.14 m
PIC time step	$6.25 \cdot 10^{-8}\ s$
Simulation duration	$0.25 \cdot 10^{-3}\ s$

- For numerical diffusion study:
  - Nodes: 51x51x251 (scenario A), 251x51x251 (scenario B)
  - Spacing: 4 cm along x and y in both scenarios, 25 mm to 25 cm along z, linearly increasing
  - FD truncation error (continuity and convergence error) increases with Hall parameter: max χ (here 35), **limited by an imposed minimum neutral background density (but I_Z tends to saturate)**
    - Background neutral density here : $2.05 \cdot 10^{18}\ m^{-3}$

Backup infos

- Comparison between the two schemes
  - Already proved that Boundary effects depend on α (*)
  - Same behavior when changing angle while comparing schemes
  - Differential boundary condition effect due to centered (FD-FV) and FW/BW schemes (FD)
  - Stronger differential effect from downstream boundary due to higher cell size there

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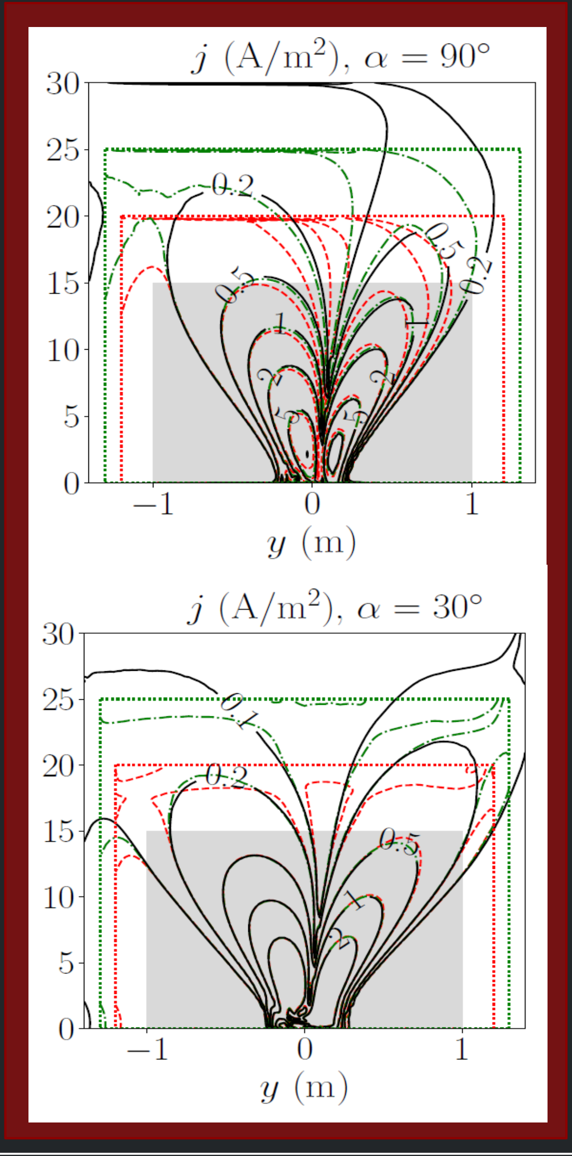
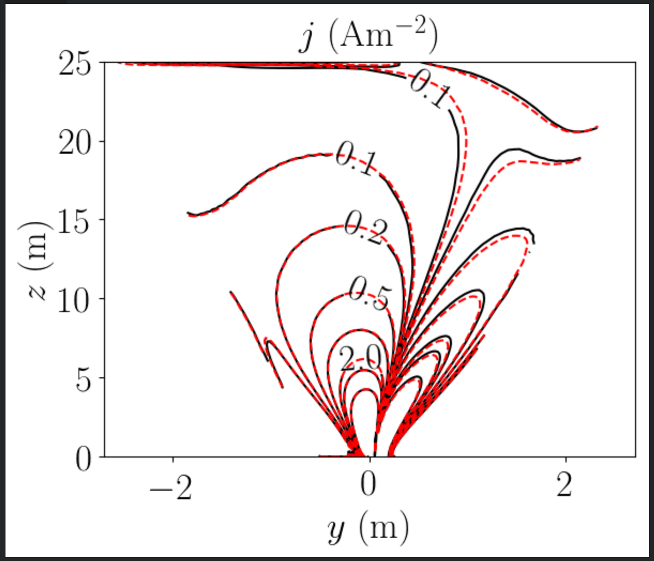
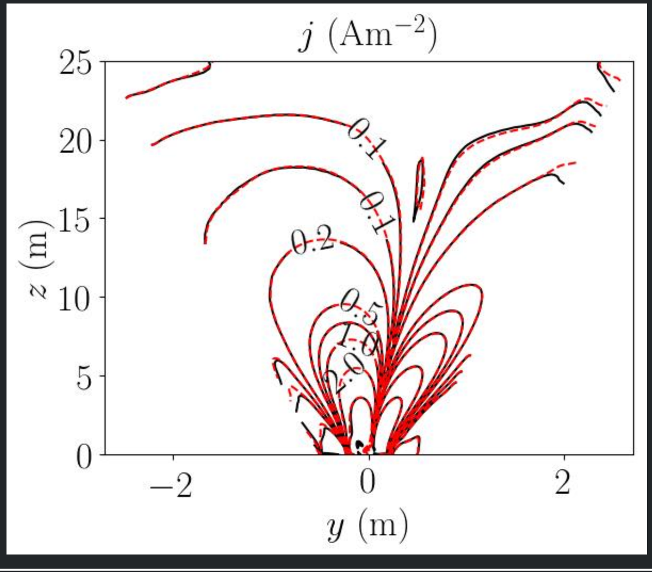




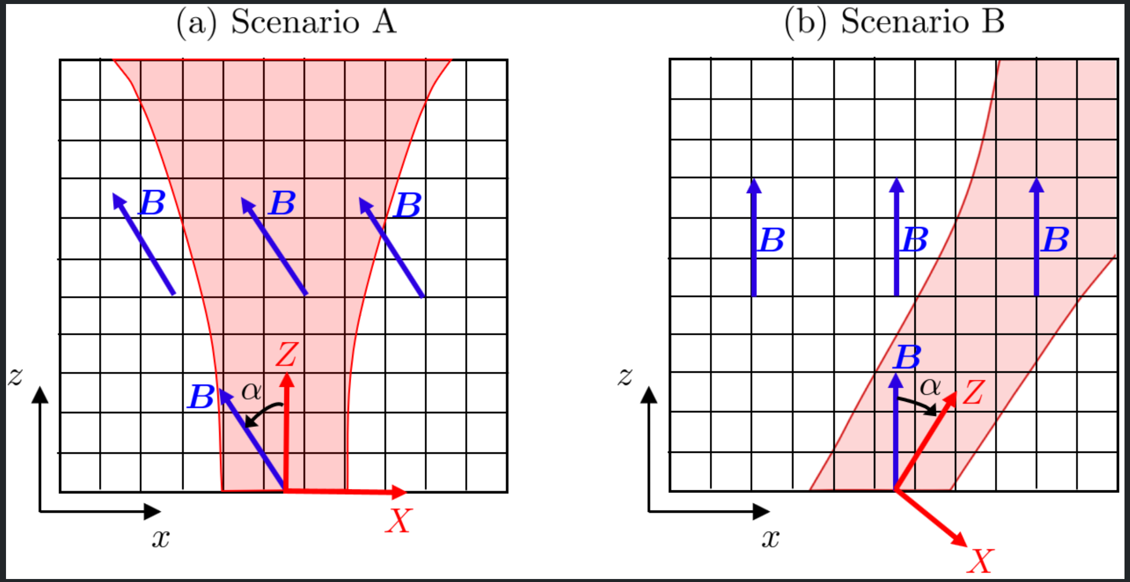


Application: numerical diffusion in magnetized plumes (I)

- Mesh directions not aligned with principal flow directions when high anisotropicity
- Diffusion tensor in mesh components has cross-diagonal terms

➢ Solutions: increase resolution, MFAM
  - Computationally expensive, complex GR algorithms, irregular cells, non-structured… really worth it?
  - Need to quantify amount of numerical diffusion effects on the solution
  - Interesting comparison between full FD scheme and hybrid FV-FD scheme numerical diffusion levels

- ASSESSMENT ALREADY DONE WITH FD SCHEME
- Simulation cases and scenarios:
  - 4 angle **physical cases** (5°, 10°, 20°, 30°)
  - 2 simulation **scenarios** (aligned and not aligned)
  - B magnitude = 0.5 G
- Oblique Self-Similar profile
  - Parks-Katz
- Meshes and settings:
  - uniform spacing along x and y, linearly increasing along z



Application: numerical diffusion in magnetized plumes (II)

- Comparison in the intrinsic plume frame $X - Y - Z$, of electric current density and its Z component at *7m*