Two-Points Power Spectral Density analysis of instabilities in PIC simulations

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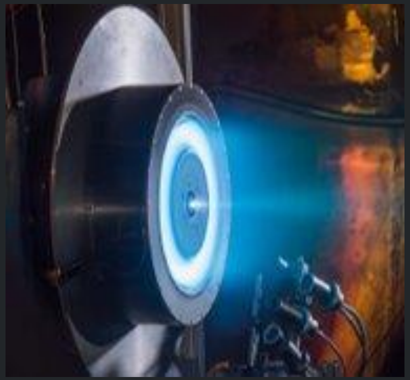
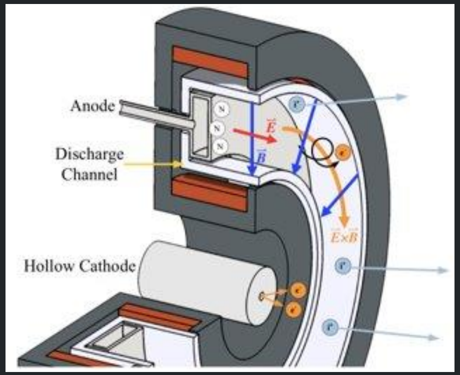
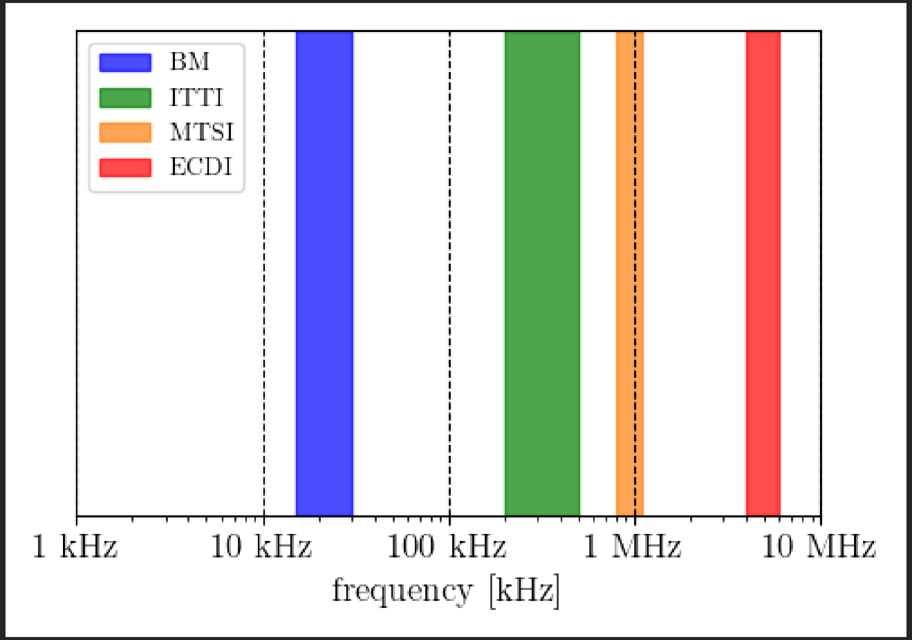
The instabilities in Hall Thrusters (HTs)







- Low frequency:
Breathing Mode (BM), predator-prey mechanism

- Mid frequency:
Ion Transit Time Instability (ITTI), related to ion ejection from thruster, mainly axial instability

- High frequency:
radial-azimuthal **Modified Two-Streams Instability** (MTSI) and almost purely azimuthal **Electron Cyclotron Drift Instability** (ECDI), **Ian Acoustic Wave** (IAW), related to the differential motion between electrons and ions

How to the detect and study the instabilities?

- **Azimuthal direction**:
  FFT techniques are good
  (because of periodicity)

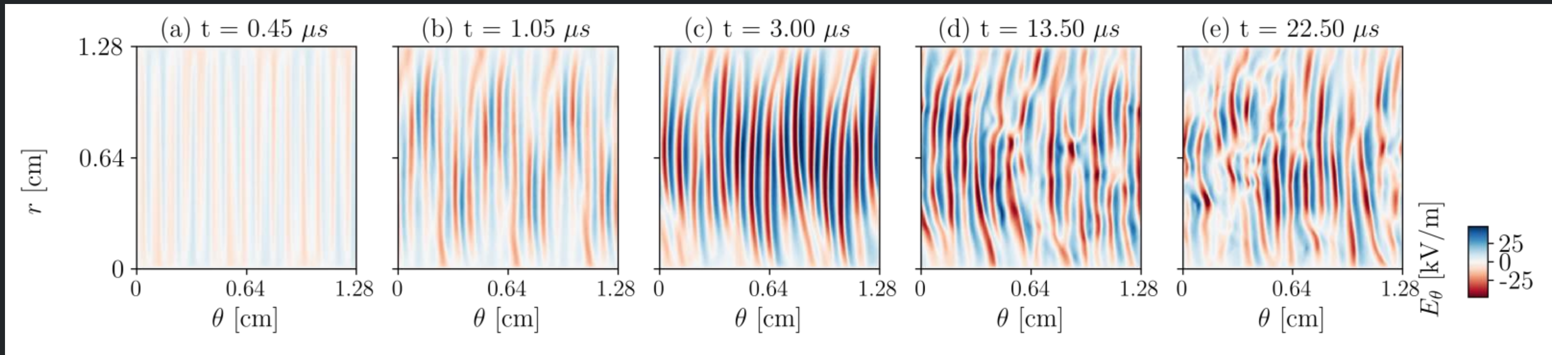


Figure obtained with a simulation as in [1]

- **Radial direction**:
  FFT with signal windowing is a valid option

- **Axial direction**:
  Too strong gradients: the standard FFT techniques are not adapted in this case, i.e. the axial direction in the axial-azimuthal benchmark case

Azimuthal: $y = \theta$
Radial: $\mathrm{z} = \mathrm{r}$
Axial: x

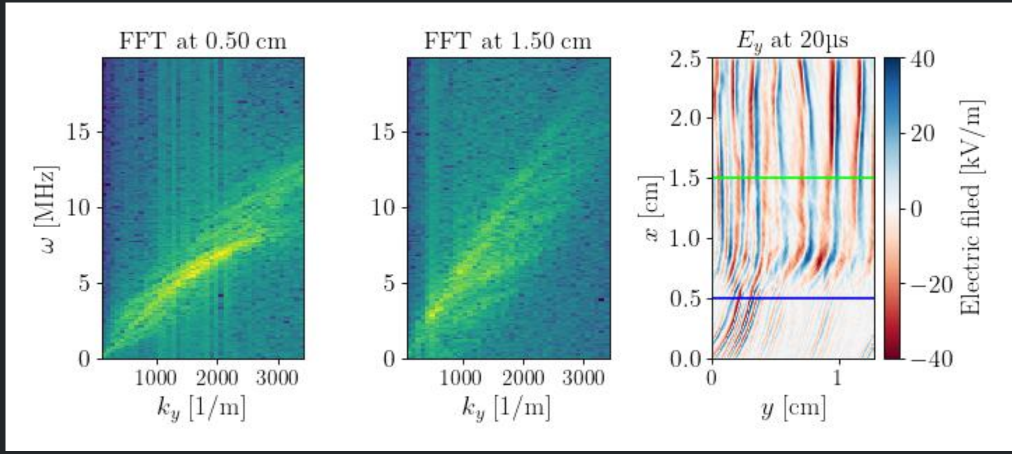


Figure obtained with a simulation as in [2]. The spectra are calculated with using FFT.

$\Longrightarrow$ Use the Two-Points Power Spectral Density Technique (PSD2P)

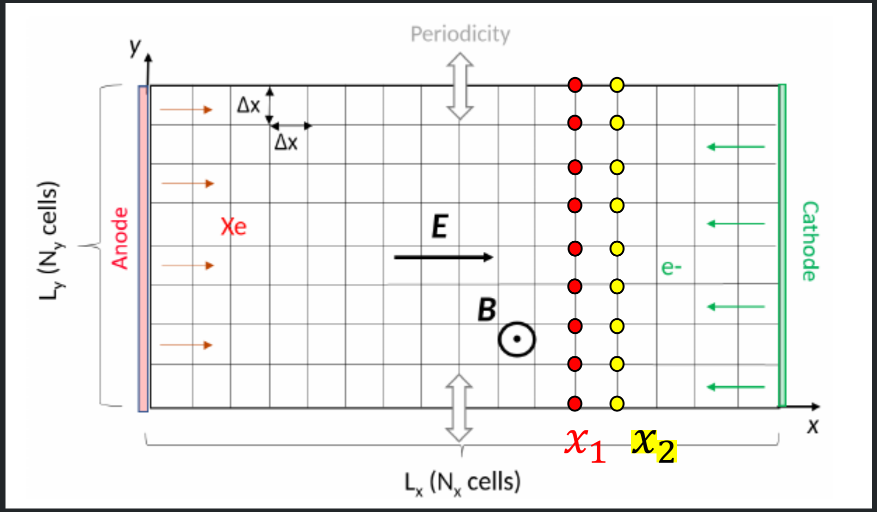
[1] W. Villafana, F. Petronio, *et al.* "2D radial-azimuthal particle-in-cell benchmark for E×B discharges", Plasma Sources Science and Technology, 30(7):075002, (2021).

[2] Charoy et al. (2019). “2D axial-azimuthal Particle-In-Cell benchmark for low-temperature partially magnetized plasmas”, Plasma Sources Sci. Technol. 28, 105010, (2019)

The Two-Points Power Spectral Density technique*

The method was developed by Beall *et al.*[3] and by Dudok de Wit *et al.*[4]

Calculate the correlation between two points and from that it can estimate the local spectrum.



For every point we calculate a **signal β** as

$$\beta(t, y_j, x_n) = n(t, y_j, x_n) - < n(t, y, x_n) >_y.$$

Temporal transform (FFT, Wavelet, …),

$$\hat{\beta}(\omega, y_j, x_n) = transf(\beta(t, y_j, x_n)).$$

The cross spectrum for the j^{th} couple is

$$P^j(\omega, y_j) = \hat{\beta}^*(\omega, y_j, x_1)\hat{\beta}(\omega, y_j, x_2) = C^j(\omega) + i\, D^j(\omega)$$

The self-correlation reads $S_n^j(\omega, y) = \hat{\beta}^*(\omega, y_j, x_n)\hat{\beta}(\omega, y_j, x_n)$, with $n = 1{,}2$ and $j = 1, \dots, M$ with M couples

Local wavenumber is given by:

$$k^j(\omega) = \frac{\arctan\left(\frac{D^j(\omega)}{C^j(\omega)}\right)}{x_2 - x_1}$$

$$dk = 2\pi/\Delta x$$

Local spectrum:

$$PSD = \frac{1}{M}\sum_{y=1}^{M} I_{[0,dk)}[k - k(\omega, y)]\, \frac{1}{2}(S_1(\omega, y) + S_2(\omega, y))$$

Indicator function

Weight of the component

*calculated with ensemble average, the version with time average is shown in the backup slides

[3] J. M. **Beall**, et al., "Estimation of wavenumber and frequency spectra using fixed probe pairs", Journal of Applied Physics, 53(6):3933–3940, June 1982.

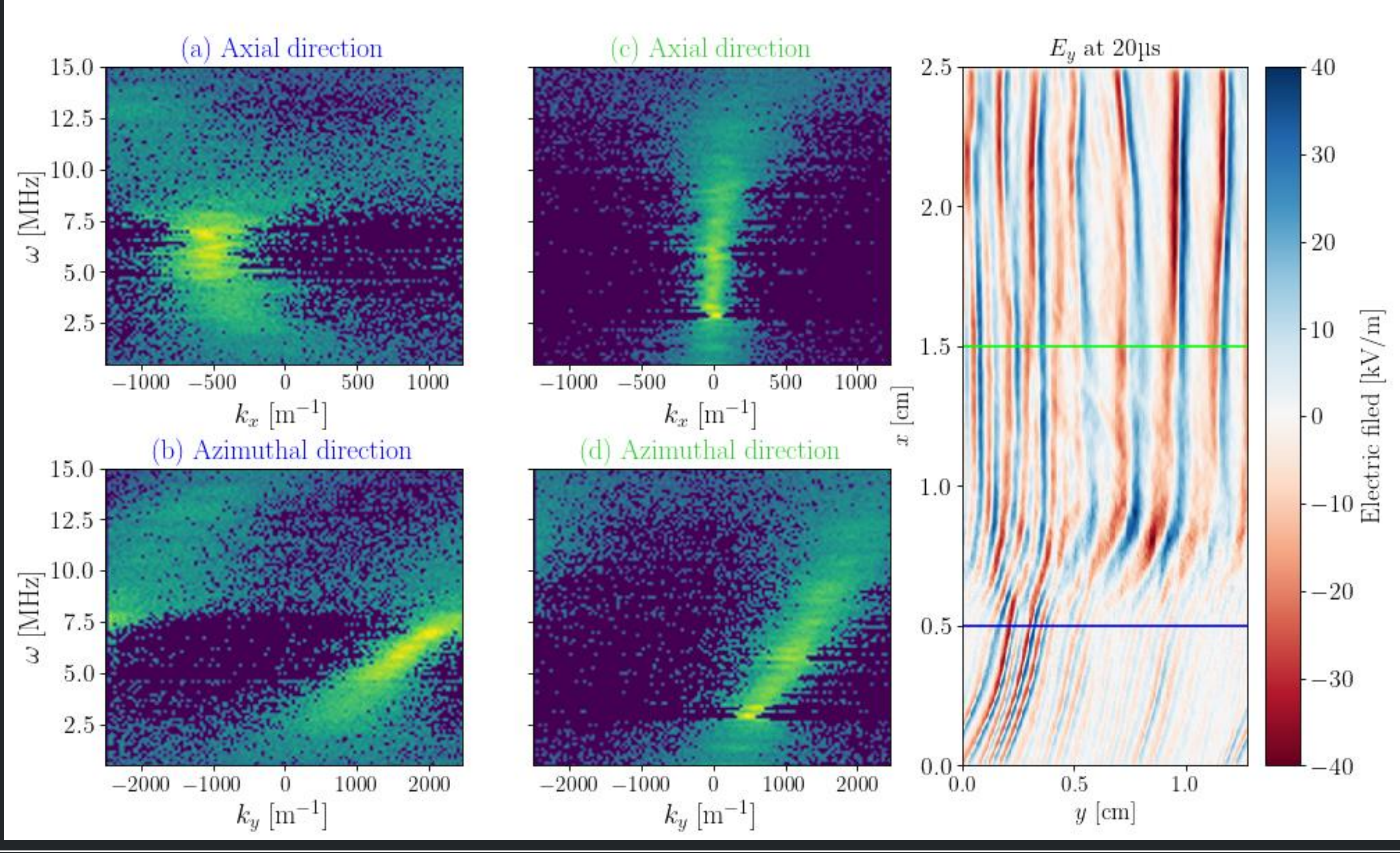
[4] T. **Dudok de Wit,** et al.. "Determination of dispersion relations in quasi-stationary plasma turbulence using dual satellite data." Geophysical Research Letters, 22(19):2653–2656, October 1995.

Application to the Benchmark case (I)

PSD spectrum calculated at two different axial position in a benchmark simulation.
Same case as the one shown in slide 3.

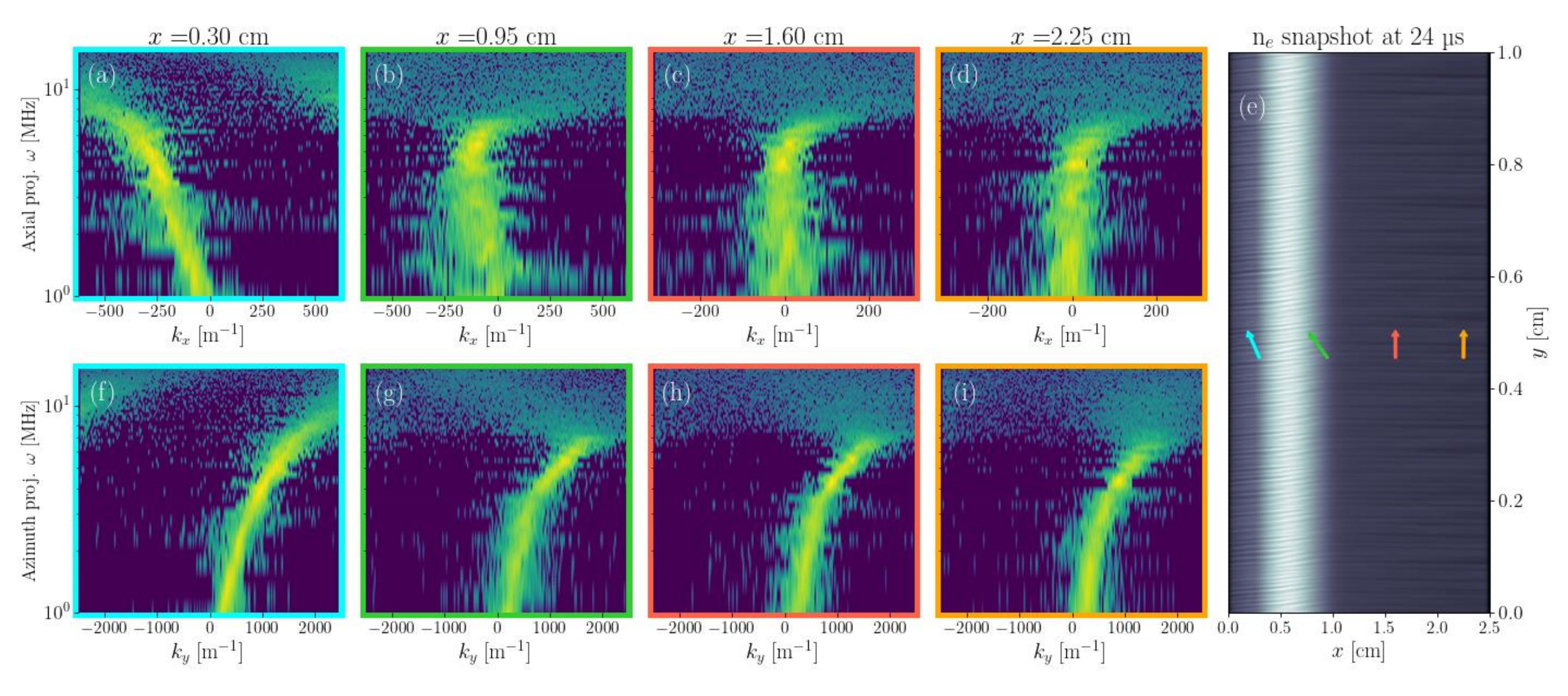
The PSD2P technique allows to reconstruct the spectrum in the axial direction at a precise axial coordinate

The shape of the instability in the azimuthal direction is very similar to the one obtained with the standard FFT technique in slide 3



Application to the Benchmark case (II)

Spectrum variation in 2D along the axis. More azimuthal points allow to have better resolved PSDs.

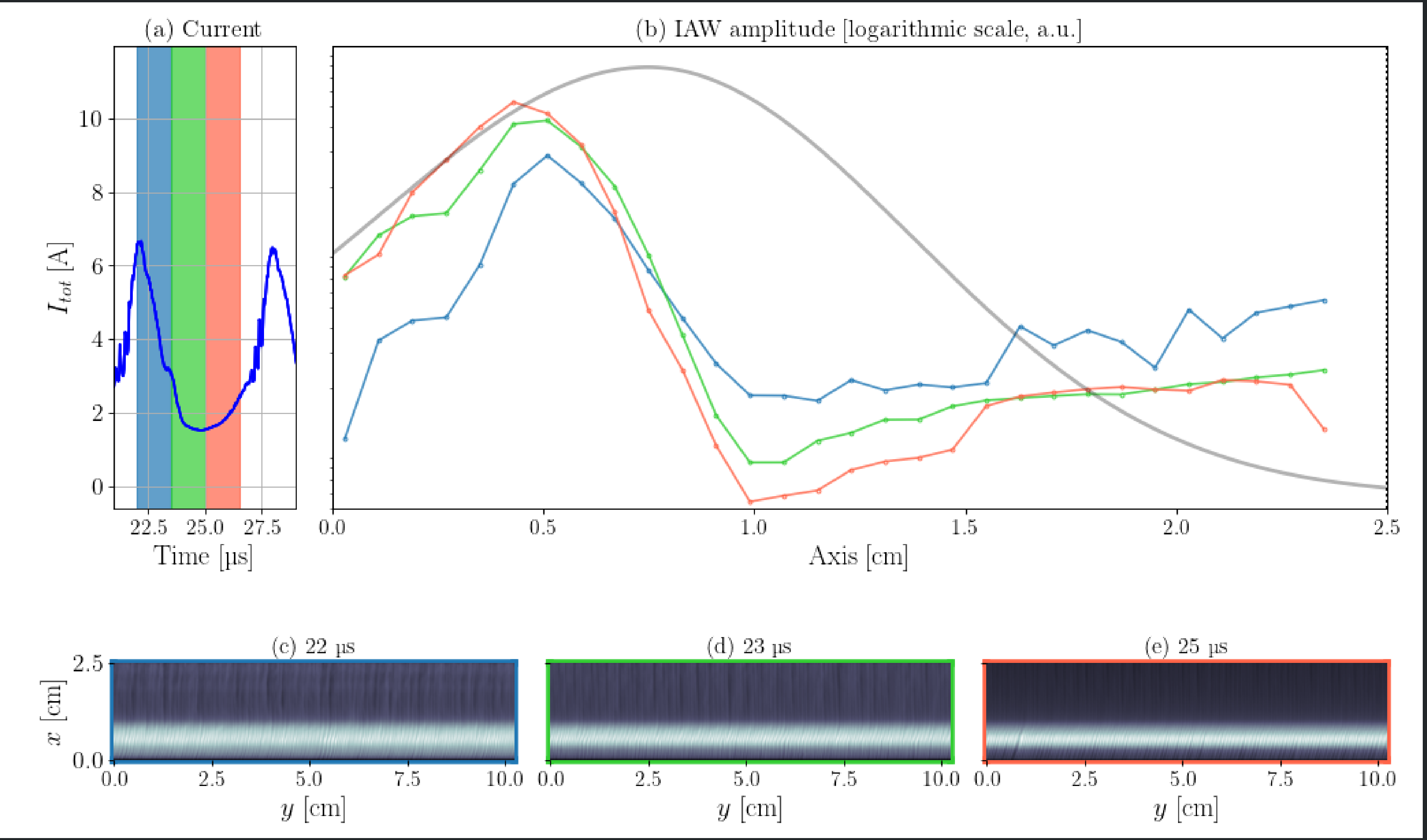


Application to the Benchmark case (III)

Amplitude of the spectrum component during different phases of the discharge, obtained by summing the axial and azimuthal components

This allow to study the variation of the axial intensity profile of the instability with respect to time

Here the Breathing Mode oscillations are quite fast, so it is difficult to apply this technique in a "steady case"



Conclusions

- The PSD2D technique allows to study the instabilities with a component directed in the axial direction, which was not possible with the standard FFT techniques
- The numerous instabilities in HTs' discharges can be easily analyzed using this technique and the maps can be compared with the analytic dispersion relations
- The amplitude evolution of each instability along the axis and along time will give important insights about the birth position and moment of the instability
- The images in these slides are obtained with an ensemble average. It is possible to reconstruct the PSD using a time averaging technique and the signal measured only in two points

Acknowledgments

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The PSD2P – **ensemble** average

Take for every point we calculate a **signal β** :

$$\beta(t,y) = \frac{n(t,y) - < n(t,y) >_y}{< n(t,y) >_y}.$$

Temporal transform (FFT, Wavelet, …),

$$\hat{\beta}(\omega, y_j, x_n) = transf(\beta(t, y_j, x_n)).$$

The cross spectrum for the j^{th} couple is

$$P^j(\omega, y_j) = \hat{\beta}^*(\omega, y_j, x_1)\hat{\beta}(\omega, y_j, x_2)$$
$$= C^j(\omega) + i\, D^j(\omega)$$

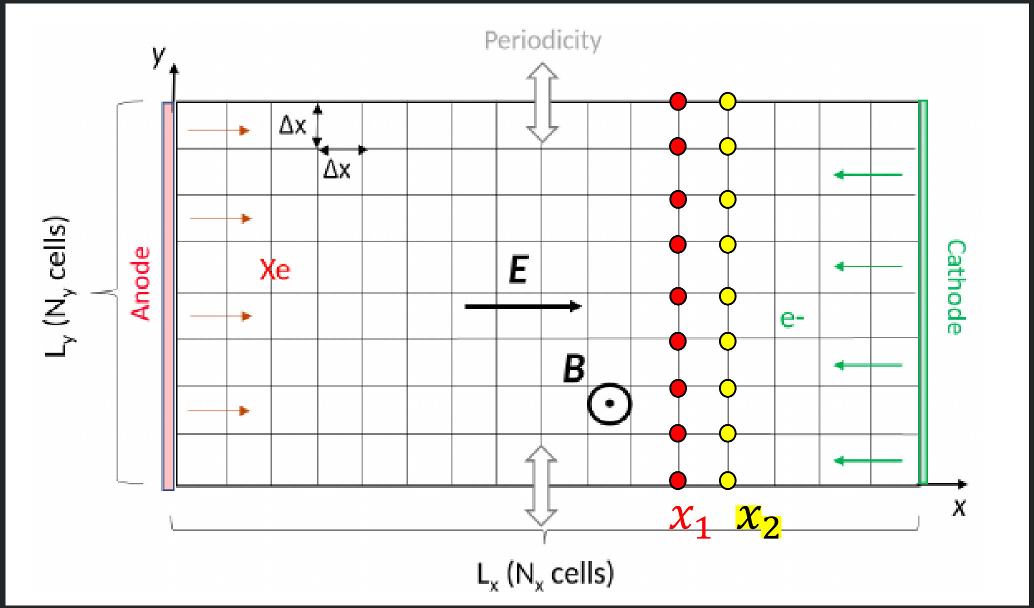
$$S_n^j(\omega, y) = \hat{\beta}^*(\omega, y_j, x_n)\hat{\beta}(\omega, y_j, x_n),$$

with $n = 1,2$ and $j = 1, \dots, M$ with M couples

Local wavenumber is given by:

$$k^j(\omega) = \frac{\arctan\left(\frac{D^j(\omega)}{C^j(\omega)}\right)}{x_2 - x_1}$$

$$dk = 2\pi/\Delta x$$



Local spectrum with **ensemble average**:

$$PSD = \frac{1}{M}\sum_{j=1}^{M} I_{[0,dk)}[k - k^j(\omega)]\frac{1}{2}(S_1^j(\omega,y) + S_2^j(\omega,y))$$

Indicator function

Weight of the component

The PSD2P – **time** average

Take for every point we calculate a **signal β** :

$$\beta(t,y) = \frac{n(t,y) - < n(t,y) >_y}{< n(t,y) >_y}.$$

Wavelet transform,

$$\hat{\beta}(a,\tau,x_n) = \int \beta(t,y,x_n) a^{-\frac{1}{2}} h^* \left(\frac{t-\tau}{a}\right) dt\,.$$

With Morlet waves

$$h(t) = \pi^{-1/4} \exp(-2\pi i t) \exp(-t^2/2)$$

and $a = 2\pi/\omega$.

We are not considering anymore y as a variable, since we have just one couple.

The cross spectrum is

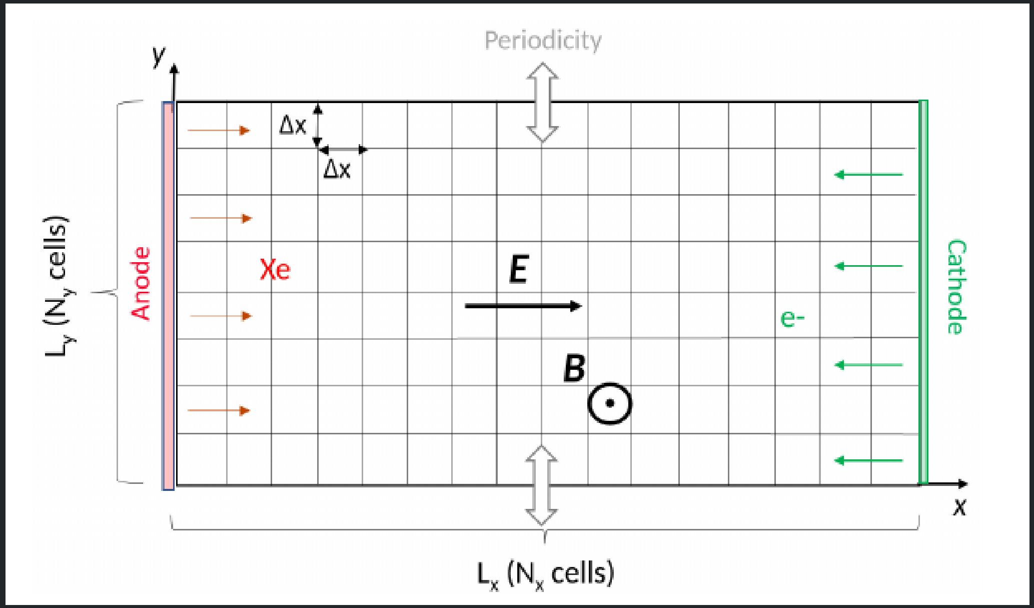
$$P(a,\tau) = \hat{\beta}^*(a,\tau,x_1)\hat{\beta}(a,\tau,x_2) = C(a,\tau) + i\,D(a,\tau)$$

Local wavenumber is given by:

$$k(a,\tau) = \frac{\arctan\left(\frac{D(a,\tau)}{C(a,\tau)}\right)}{x_2 - x_1}$$

And the self-correlation:

$S_n(a,\tau) = \hat{\beta}^*(a,\tau,x_n)\hat{\beta}(a,\tau,x_n)$, with $n = 1,2$



Local spectrum with **time average**:

$$PSD = \frac{1}{\tau_{tot}} \int d\tau I_{[0,dk)}[k - k(a,\tau)](S_1(a,\tau) + S_2(a,\tau))$$

Indicator function

Weight of the component