Controlling perpendicular electric fields with biased electrodes

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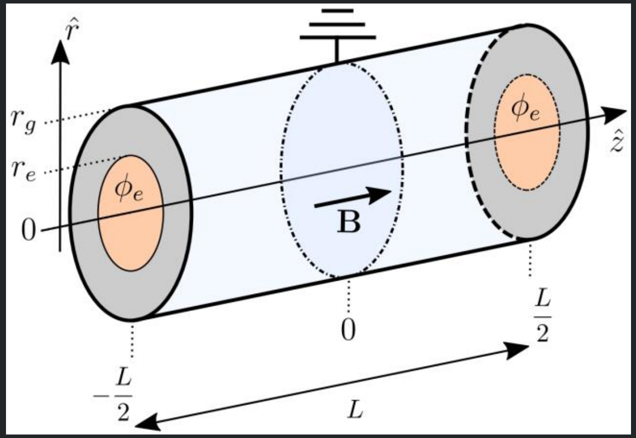
ExB Plasmas Workshop 2022
Madrid, online event

Introduction

The ability to impose and control an electric field perpendicularly to magnetic surfaces in magnetized plasmas is of importance to a broad range of applications [1]. It has notably been shown in recent years that plasma rotation in $E \times B$ configurations could offer opportunities to develop new plasma separation techniques [2], as proposed for instance for nuclear spent fuel reprocessing and rare earth elements recycling [3].



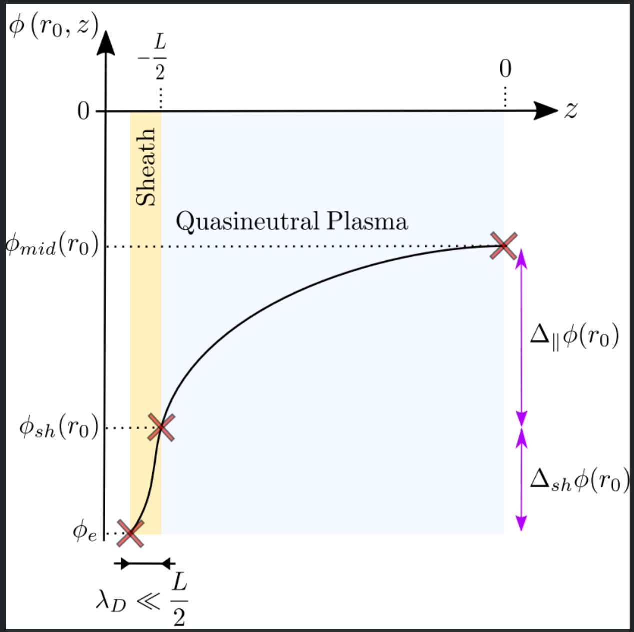
A key step to advance rotating plasma technologies is to understand how the bias imposed on an electrode

1. is transferred to the plasma through the sheath [5]

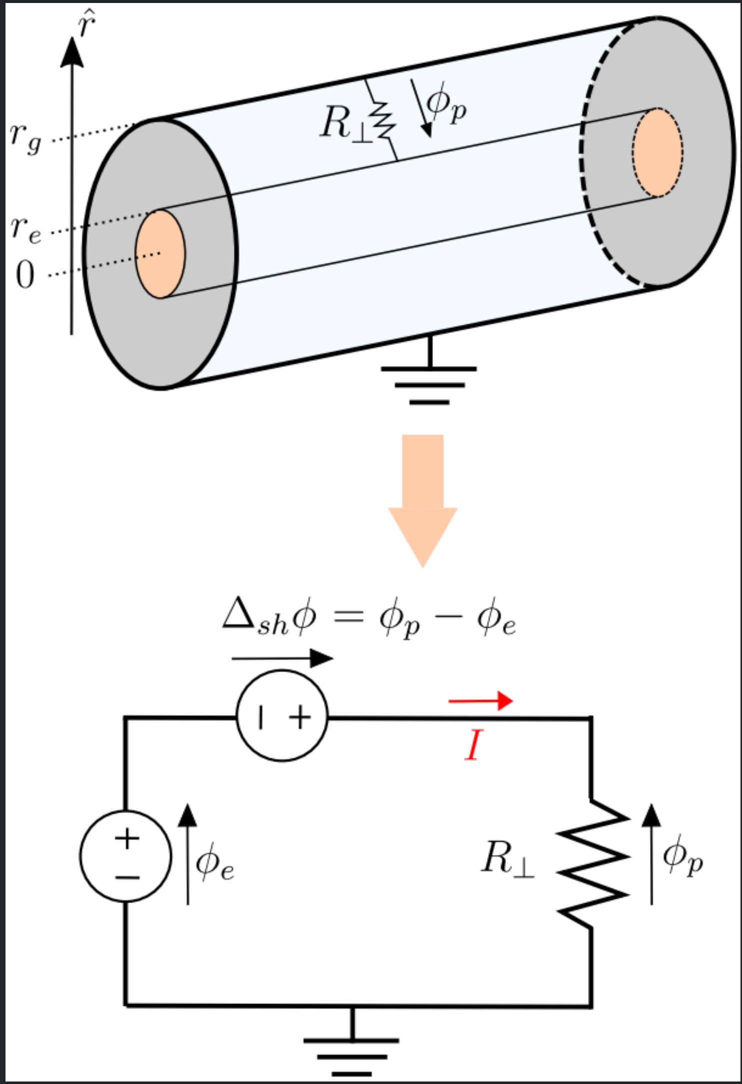
$$\Delta_{sh}\phi(r) = \phi_e - \phi_{sh}(r)$$

2. distributes itself in a magnetized plasma column [4]

$$\Delta_{||}\phi(r) = \phi_{sh}(r) - \phi_{mid}(r)$$



Discharge model



Liziakin *et al.* [5] proposed to model the effect of the sheath on the plasma potential at the sheath edge through an equivalent electric circuit. Three ”ingredients”

- Plasma resistance at radius r

$$dR(r) = \frac{dr}{\pi L \sigma_\perp r} \Rightarrow R_\perp = \int_{r_e}^{r_g} dR(r)dr$$

- Discharge current

$$I = \frac{V_p}{R_\perp}$$

- Current balance at electrode $I = I_e - I_{is} - I_{eth}$

$$I = I_{is}\left[\exp\left(\Lambda + \frac{V_e - V_p}{T_e}\right) - 1 - \frac{I_{eth}}{I_{is}}\right]$$

Current conservation then leads to the transcendental equation

$$\exp\left(\Lambda + \psi_e - \psi_p\right) - 1 - \Xi - \chi\psi_p = 0$$

Normalization:

$$\psi_e = \frac{V_e}{T_e}, \psi_p = \frac{V_p}{T_e}, \Xi = \frac{I_{eth}}{I_{is}}$$

Note that this model assumes a constant potential for $r < r_e$, that is a voltage drop between r_e and r_g only. For a refined model see [6,7]

Findings: two distinct operating regimes

From this transcendental equation

$$\exp\left(\Lambda + \psi_e - \psi_p\right) - 1 - \Xi - \chi\psi_p = 0$$

we identify two asymptotic regimes:

- Non-saturated: $I \ll I_{is} + I_{eth}$

$$\psi_p = \phi_e + \Lambda - \ln(1 + \Xi)$$

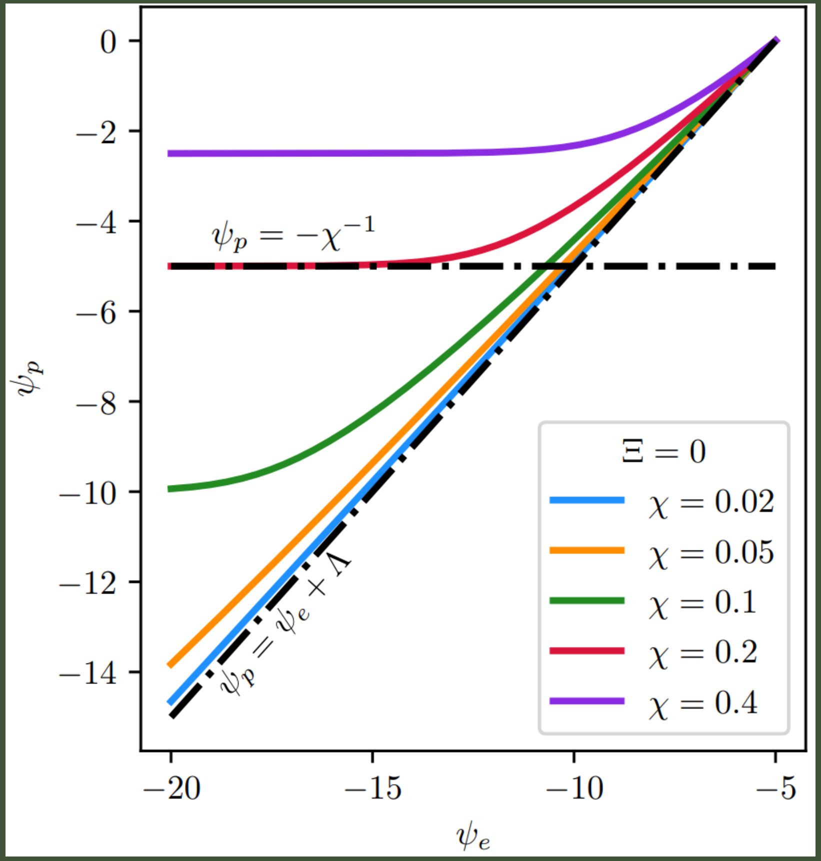
- Saturated: $I = I_{is} + I_{eth}$

$$\psi_p = -\frac{(1+\Xi)}{\chi}$$

where χ is a measure of the admissible plasma potential

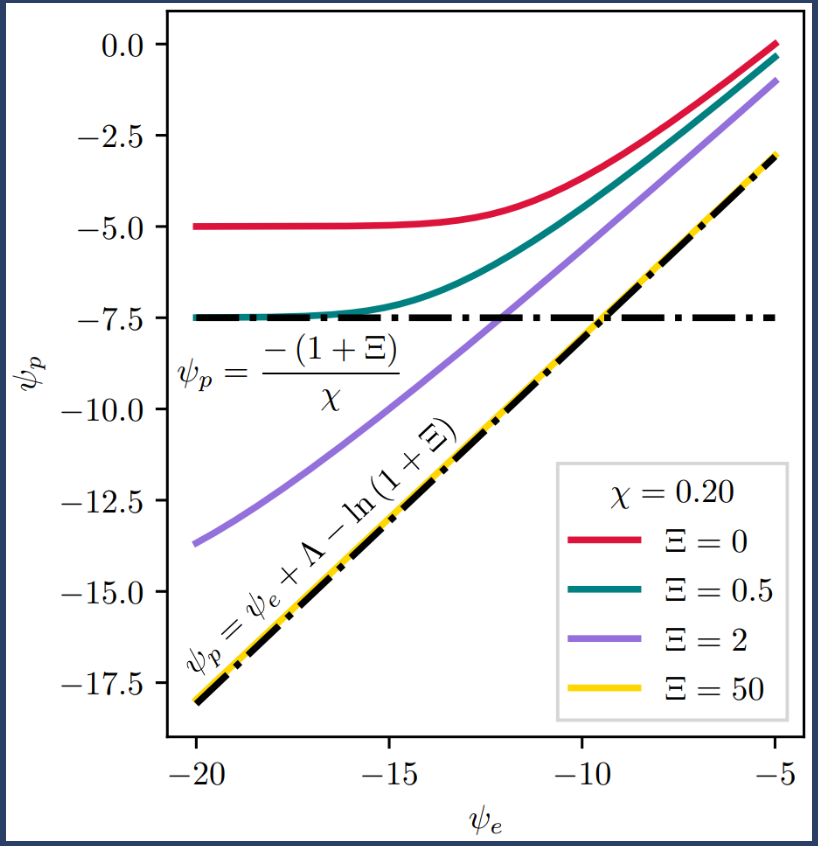
$$\chi = \frac{T_e}{I_{is}R_\perp}$$

Without thermionic emission, $\Xi = 0$



ψ_p as a function of the electrode bias for different value of χ.

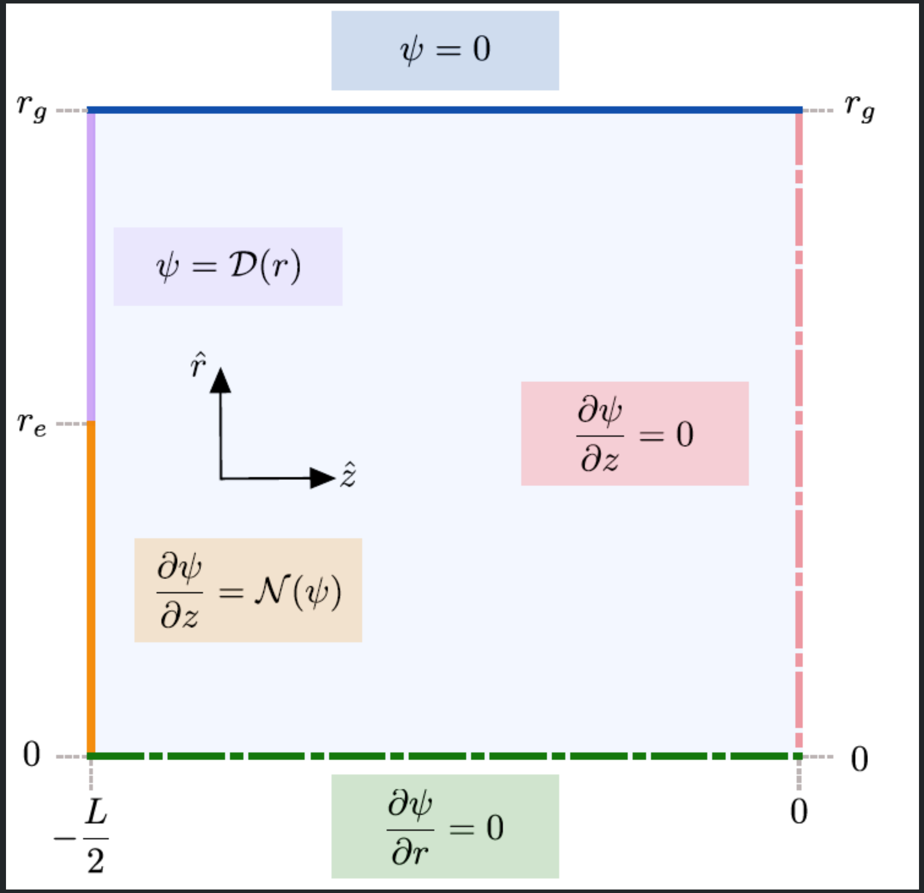
In increasing the discharge current thermionic emission (Ξ) can offer additional control.



ψ_p as a function of the electrode bias and by increasing thermionic emission, $\chi = 0.20$.

Simulation to capture the sheath and quasi-neutral plasma consistently

Solve $\nabla \cdot (\underline{\underline{\sigma}}\, E) = 0$ *over the 2D cylindrical domain*



The quasi-neutral plasma is modelled through its parallel and perpendicular conductivity, which combined with charge continuity yields in steady state the anisotropic Laplace equation

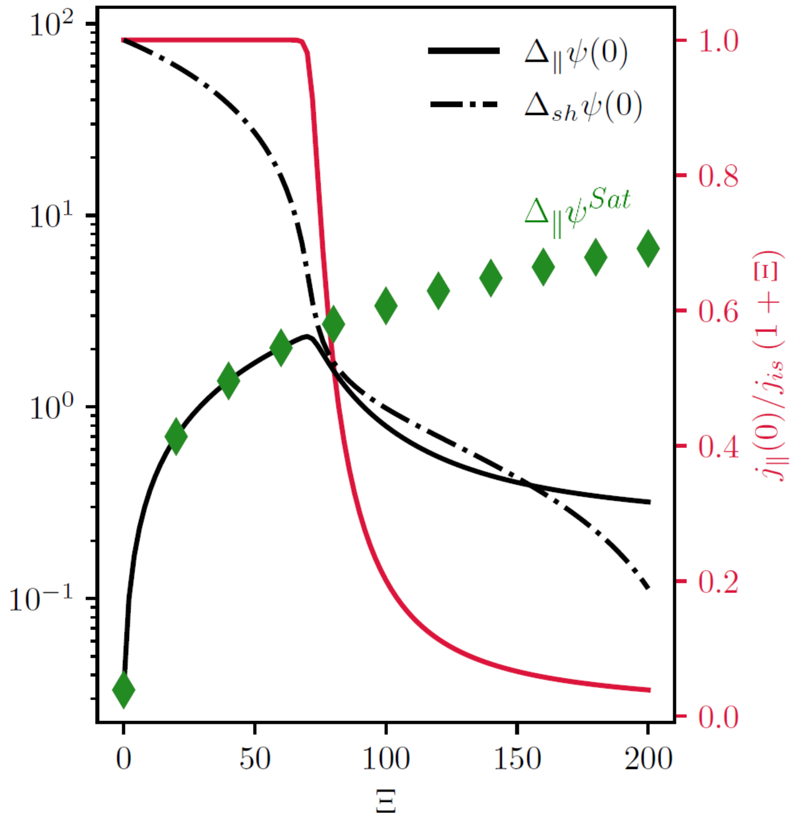
$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial \psi}{\partial r}(r,z)\right] + \frac{\sigma_\parallel}{\sigma_\perp}\frac{\partial^2 \psi}{\partial z^2}(r,z) = 0 \quad \text{with} \quad \psi(r,z) = \phi(r,z)/T_e$$

The ion-sheath is modelled via a non-linear Neumann condition

$$\mathcal{N}(\psi) = \frac{j_{is}}{T_e \sigma_\parallel}\left[1 + \Xi - \exp\left(\Lambda + \psi_e(r) - \psi(r, z_0)\right)\right]$$

This system is solved numerically via finite difference discretization.

Voltage drop along field lines on axis: thermionic emission



Evolution of the on-axis voltage drop $\Delta_{\|}\psi(r=0)$ with increasing thermionic current (Ξ).

We verify that, consistent with earlier theory

- Plasma potential in saturated regime

$$\psi_p \propto -\Xi$$

- Voltage drop across the sheath:

$$\Delta_{sh}\psi \propto |\psi_e| - \Xi$$

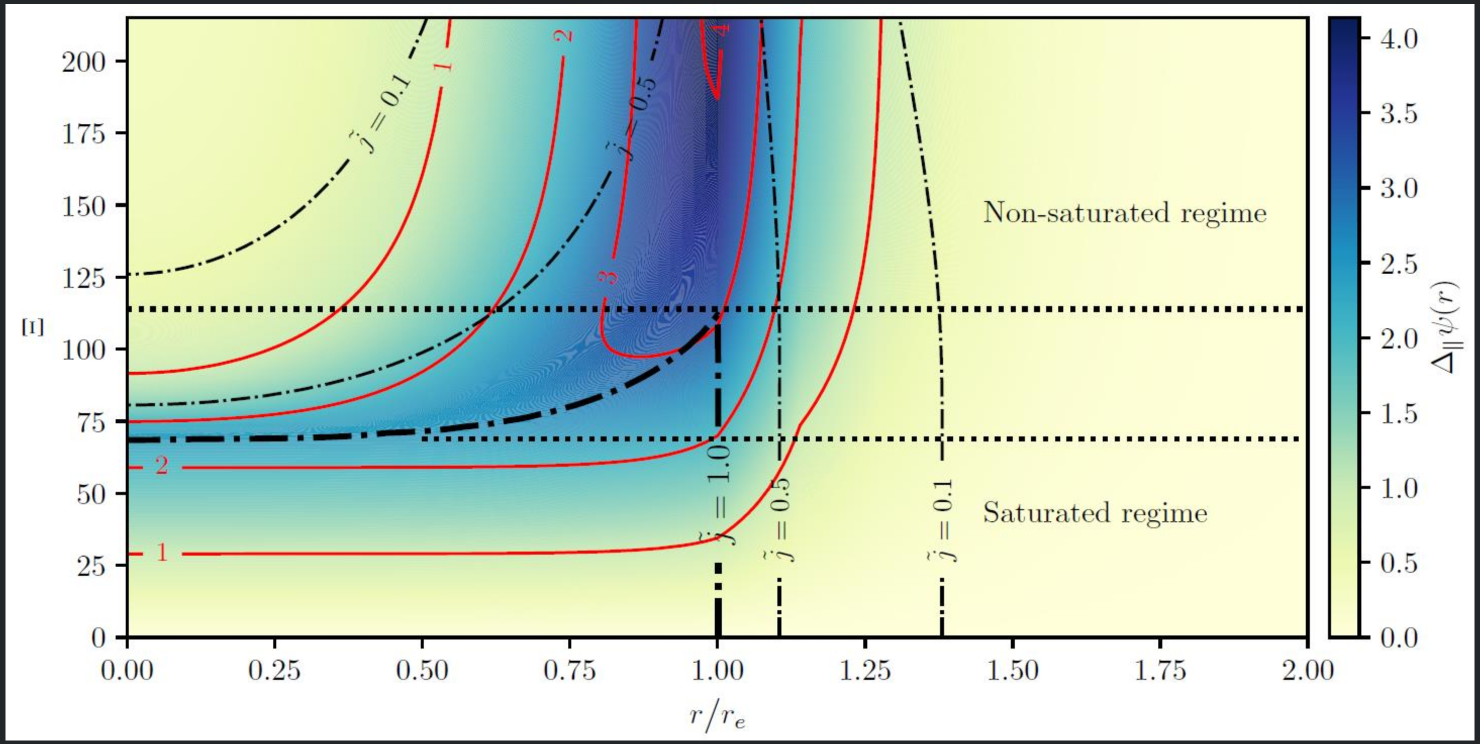
We also observe that the voltage drop along field lines in the saturated regime

$$\Delta_{\|}\psi \propto \Xi$$

This last result can be derived analytically from the saturated current and Ohm's law at the sheath, to get

$$\Delta_{\|}\psi^{Sat} = \frac{L}{4T_e\sigma_{\|}} j_{is}\,(1+\Xi)$$

Voltage drop along field lines on axis: variation with radius



Voltage drop along field lines as a function of radial position (for $r \in [0, 2 \cdot r_e]$) and for increasing thermionic emission (Ξ). Black dashdot lines indicate iso-contours of normalized current density $\tilde{j} = |j_{sh,||}|/[j_{is}(1+\Xi)]$.

Findings:

- $\Delta_{||}\psi(r)$ depends on radius
- $\Delta_{||}\psi(r)$ can be significant compared to T_e

The **saturated/non-saturated regime** is a **local phenomenon** : s-> ns transition starts at the outer edge of the biased electrode and progressively moves radially inward until full saturation has been reached [6] .

The density current drawn at a given electrode radius controls the voltage drop along field lines [7].

Conclusions

- The plasma potential cannot be arbitrary controlled by imposing a negative bias on an electrode.
- Simple electric considerations show it is a function of the resistance across the radius of the plasma column as well as of the ion saturation current. Scaling laws can then be derived from conductivity.
- Thermionic emission, by increasing the current though the plasma column,
  1. allows greater control over the plasma potential at the sheath edge compared to the case of cold electrode, but
  2. is responsible for an increase of the plasma potential drop along magnetic field lines in the quasi-neutral plasma.

For more information on this work, please see Ref.[7] and/or write us (*baptiste.trotabas@laplace.univ-tlse.fr, renaud.gueroult@laplace.univ-tlse.fr*)

References

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